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AN AXISYMMETRIC NEAR WAKE ANALYSIS USING ROTATIONAL CHARACTERISTICS

by

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POLYTECHNIC INSTITUTE OF BROOKLYN

DEPARTMENT
of
AEROSPACE ENGINEERING
and
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SUMMARY

The near wake of a cone in a hypersonic stream is analyzed by simultaneously solving the inviscid region and the viscous shear layer.

The inviscid region is solved by the use of rotational axisymmetric characteristics. It is assumed that viscosity and heat transfer play an important role only within a region bounded by streamlines which at the trailing edge of the cone are for the most part in the subsonic portion of the boundary layer. This region, termed the shear layer, lies between the Dividing Streamline (or centerline) and the Basic Streamline. The solution to the inviscid region is obtained by specifying conditions along the characteristic line originating at the shoulder of the cone, and by specifying the pressure distribution along a free surface (Basic Streamline) taken to be the streamline which at the shoulder of the cone separates

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the supersonic from transonic and subsonic portions of the boundary layer. The pressure distribution along the Basic Streamline is iterated until the mass flow, momentum, and energy in the shear layer are consistent with the location of the Dividing Streamline and with the initial conditions at the edge of the cone.

Profiles for pitot pressure, static pressure and stagnation enthalpy are presented and compared with experiments at different downstream locations. The shape and strength of both the lip and recompression shock are also shown. Both sets of results are seen to be in very good agreement with the experimental results available.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Contd)

LIST OF SYMBOLS

A	=	$\cot\mu / V$, coefficient in characteristic equation
$\bar{A}$	=	Area of each "n" strip
B	=	$\sin\mu \sin\theta / y\cos(\theta+\mu)$
b	=	$\tan(\theta - \mu)$
c	=	$\sin 2\mu / 2\gamma\bar{R}$
C_f	=	coefficient of friction
d	=	$\sin\mu \sin\theta / y\cos(\theta - \mu)$
D	=	base diameter of cone
e	=	$\tan\theta$
g	=	$\tan(\theta + \mu)$
H	=	stagnation enthalpy
h	=	H/H_∞
J	=	o for two dimensional flow; 1 for axisymmetric flow
K	=	coefficient of thermal conductivity
M	=	Mach number
m	=	$\sin\mu / \cos(\theta - \mu)$
N	=	Dimension in direction normal to a streamline
n	=	$\sin\mu / \cos(\theta + \mu)$
P	=	Pressure
p	=	p/p_∞
Q	=	Heat transfer rate
q	=	$Q/\pi R^2 \rho_e V_e, H_e$ non-dimensional heat transfer rate
R	=	Base radius of cone
$\bar{R}$	=	gas constant
Re	=	Reynolds number

S	=	entropy
s	=	$(s - s_{\infty})/\overline{R}$
T	=	$\cos\mu/\cos(\theta + \mu)$
V	=	local velocity
V_L	=	$\sqrt{2H}$, local limiting velocity
v	=	V/V_L
X	=	coordinate in streamwise direction
x	=	X/R
Y	=	coordinate in direction normal to x axis
y	=	Y/R
z	=	$\cos\mu/\cos(\theta - \mu)$
θ	=	local flow deflection
μ	=	$\sin^{-1} \frac{1}{M}$
$\overline{\mu}$	=	coefficient of viscosity
ρ	=	density

subscripts

$(\ )_{\infty}$	=	free stream conditions
$(\ )_b$	=	values at base of cone
$(\ )_e$	=	conditions at edge of boundary layer at trailing edge of body
$(\ )_i$	=	conditions along strip number "i"
$(\ )_t$	=	stagnation values
$(\ )_w$	=	values on conical surface
$(\ )_1, (\ )_2$		conditions at a known point from which a characteristic line emanates
$(\ )_3$		conditions at a point which are found by using points 1 and 2.

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I. INTRODUCTION

The hypersonic wake of both blunt and slender bodies has received considerable attention within recent years; an overall review of the problem may be found in references 1 and 2. Many problems associated with the far wake have been analyzed so that the interest has now shifted to the solution of the near wake.

Chapman[3] analyzed the problem of mixing of a uniform stream with a semi-infinite stagnant region. However, it was not until later that the results of this basic mixing study were used to analyze the recirculation region and the shear layer behind a blunt-based body. In the mixing process, a dividing streamline is obtained which separates the fluid initially at rest from that initially in motion; this streamline, when used in conjunction with the actual body geometry, is assumed to divide the recirculation from the external flow region in the wake problem. Denison and Baum[4] later improved this analysis by solving the same problem with an initial (Blasius) boundary layer profile, which more closely approximates the actual flow conditions.

Few exact solutions have been found to the flow in the entire base region; one of these is by Viviand and Berger[5], which is valid for very low free stream Reynolds numbers. Their solution was obtained by applying Oseen's approximation to the complete equations of motion. Exact solutions for laminar flow at higher Reynolds numbers do not as yet exist.

Lees and Reeves[6] have attacked the near wake of a blunt body by the use of the integral form of the differential equations, as it was done by Crocco-Lees[7], and by reverse flow solutions to the Falkner-Skan equation. The final

form of the differential equations is obtained from the x-momentum and from the first moment of the x-momentum equation. It is assumed that mixing takes place at constant pressure so that the equations are simplified into two ordinary differential equations in two unkowns (velocity on centerline and displacement thickness). Once the calculation is carried to the rear stagnation point then a new set of ordinary differential equations is used (pressure is now allowed to change). This new set of equations is now solved the same way as the previous ones. It turns out that for a given family of solutions there will be only one set of values which will enable the calculation to go downstream (past the critical point). For any other values a second stagnation point or zero pressure on the centerline is obtained. The inviscid flow field may be assumed to be governed by the Prandtl-Meyer equations.

Due to the vorticity created by the sudden expansion of the flow at the base of a blunt based slender body in hypersonic flow, the above theory cannot be applied to this class of problems. Reeves and Buss[8] have analyzed this problem by using the equations of Lees and Reeves for the region downstream of stagnation point while upstream of it the Navier-Stokes equations are solved by a double Taylor series expansion in the stream function and flow variables about the rear stagnation point. A seventh degree series is used and the coefficients are determined by the symmetry conditions along the axis, the boundary conditons and temperature along the base of the body and the Navier-Stokes equations. The outer inviscid flow is solved by the method of streamtubes. This method may be applied to two-dimensional or axisymmetric bodies.

Rom[9] and Rom and Victor[10] have used a modified form of the Crocco-Lees technique and with the help of semi-empirical results have been able to correlate experimental results. Webb, Golik, Vogenitz and Lees[11] have also extended the analysis of Crocco and Lees and have obtained results downstream of the rear stagnation point by applying polynomials in a two moment (momentum equation) plus one moment (energy equation) calculation and also in a three moment plus a two moment system. This system of resulting equations permits more degrees of freedom in the choice of the inital profiles.

All the theories discussed above do not lead to detailed solutions but only give overall characteristics of the flow field (velocity on centerline, displacement thickness, momentum thickness etc). Weiss[12] and Baum and Denison[13] are the first ones to have analyzed the flow field in detail. Weiss' analysis is limited from the trailing edge to the rear stagnation point while Denison and Baum have attacked the problem by starting at the rear stagnation point. From trailing edge to rear stagnation point, the flow is split up in three regions (outer flow, a shear layer, recirculation region). The outer flow is solved by the method of characteristics. The shear layer is analyzed by a linear approximation of the boundary layer equations (Oseen's approximation) while the recirculation region is solved by the inviscid Navier-Stokes equations in terms of vorticity and stream function. The solution for the recirculation region is obtained by assuming a temperature distribution from which a velocity distribution is arrived at, which in turn is used to solve the energy equation for a new temperature and pressure distribution. Baum and Denison commence their analysis at the rear stagnation point and integrate the equations by an implicit

difference scheme. The equations which are considered are the continuity equation, the x-momentum and energy equations in the boundary layer form and the y-momentum equation as applicable to an inviscid flow. The equations are then integrated in the Von-Mises coordinates. However, since the resulting equations for the x-derivatives would have a singular point at $u=a$ and would be unstable for $u<a$, the transverse momentum equation for $u<a$ is replaced by the statement that p is not a function of the stream function. (This replacement forces a physically non-existent saddle point on the solution). Now as soon as any family of initial profile is picked, only one profile within a given family may permit us to go through the critical point. Any other profile (as it happens in Lees-Reeves theory) will give zero pressure or a second stagnation point somewhere along the centerline. As explained by Weinbaum[14] Baum and Denison wrongly feel that if no eigenvalue to the particular family of profiles exist, or if two eigenvalues exist then the problem either is not posed correctly or the steady state solution as obtained from the unsteady equation would have to be analyzed.

Weinbaum recently has critically examined the differential equations (boundary layers) which have been used by previous authors. He has concluded that the critical point obtained by most investigators is only an artificial way by which the equations used (boundary layer) manifest themselves as not having a dynamic adjustment at the throat (i.e., when boundary layer equations are used v at the outer edge of the viscous region cannot be arbitrarily specified, and one has to accept whatever it turns out to be). Not only is the critical point artificial, but its location may be varied at will (within bounds dictated by parameters) by suitably choosing

different positions for the edge of the viscous region. The equations which he considers are the same as the ones of Baum and Denison, except that the transverse momentum equation is retained in the subsonic region. The point u=a now requires special care due to the fact that the derivatives in the x direction will be in an indeterminate form which may be evaluated by l'Hopital's rule. With this new set of equations no eigenvalue problem is encountered and any arbitrary set of stagnation point profiles will be able to pass downstream. The correct solution will then be obtained when the ambient pressure is recovered at the end of strong interaction region. The Rudman-Rubin[15] equations are similar to the ones used by Weinbaum with the exception that the x derivative of the pressure term has been neglected. This minor difference causes a major breakthrough in the solution, because the singularity at u=a has now disappeared and the numerical technique used may be simplified considerably.

The flow field described above is not amenable to a single solution unless the complete Navier-Stokes equations are utilized. A solution can also be obtained by splitting the problem into the following four distinct flow regimes: 1) leading to trailing edge of body, 2) expansion of fluid at the trailing edge, 3) trailing edge to rear stagnation point 4) rear stagnation point to downstream infinity. For the sake of simplicity it has to be assumed that no interactions between the different regions take place.

If the flow is assumed to be laminar from the leading to trailing edge of the cone, then the solution can be obtained for either of the two conditions. For large Reynolds numbers the inviscid flow field is solved by either the method of characteristics or by solving the inviscid conical

flow equations. Once the inviscid field is known then the thin viscous layer around the body can be solved by the usual boundary layer techniques. For low Reynolds numbers the problem is more complicated because no distinct viscous layer, shock wave and inviscid regions exist and the viscous layer (even to a first approximation) cannot be neglected with respect to the inviscid region.

The expansion of the rotational flow at the trailing edge is a problem which is now being studied. This problem is complicated by the upstream influence of the base pressure through the subsonic portion of the boundary layer. This problem has been investigated by Weinbaum[16] for incompressible flow, Baum[17] and Weiss and Nelson[18] for supersonic high Reynolds number flows. In all the cases mentioned, no interaction between the boundary layer and inviscid flow field is assumed.

For the analysis to proceed from the trailing edge to the rear stagnation point, the recirculation region, shear layer and the outer inviscid flow would have to be solved independently remembering that the boundary conditons connect the three solutions together (obviously for low densities, this procedure cannot be adapted because of the interaction problems involved). The recirculation region and the shear layer can best be analyzed by the methods developed by either Weiss[19] or Moretti[20]. Weiss' method while not as detailed as the approach used by Moretti, has the advantage of being soluble within a short period of time. In both cases the inviscid outer flow is solved by the method of characteristics. Moretti has numerically solved a modified form (viscosity is retained while for simplicity heat transfer is neglected) of the unsteady Navier-Stokes equations. Due to the hyperbolic nature of the equations a Lax-

Wendroff technique is used to obtain the solution. The steady state solution is then assumed to be the asymptotic time limit of an unsteady flow field. By using these equations, both the recirculation and shear layer region can be solved. With slight modifications the equations could also be used for low density flows. However, the usefulness of this method is offset by the enormous time required to obtain a solution (on an IBM 7094 the time for an accurate calculation would be of the order of several hours.)

A semi-empirical approach which can be worked out with the aid of the method of characteristics and the equations used by Weinbaum and Garvine[21] or the ones originally analyzed by Rudman and Rubin will now be outlined. The main tool to be used in this analysis will be the method of characteristics. Application of the method of characteristics to the near wake was first suggested by M.H. Bloom in a presentation at an I.D.A. Conference* in 1963. Calculations showing the importance of radial pressure gradients and the thickening of the shear region (after the expansion of the surface boundary layer) immediately downstream of the shoulder of an axisymmetric body were also shown by Bloom and Vaglio-Laurin[22]. The first published results of this method applied to the near wake problem are by Weiss,[12] Weinbaum[29,23] and Weiss and Weinbaum[24]. In reference 12, the base region of the flow over a wedge is treated and an approximate solution is obtained by matching the free shear layer, recirculation, and inviscid flow regions. The assumptions of both Chapman, and Denison and Baum that the stagnant (recirculating) region is semi-infinite is no longer necessary and thus the effect of finite

* The proceedings of these meetings are unpublished.

base diameter is obtained.

In reference 29 the variation of the entropy within the boundary layer was studied and in reference 23, preliminary analyses for characteristic calculations are initiated. It is also shown that for high "inviscid" Mach numbers ($M_e > 8$), less than half the total free stream expansion occurs in the centered expansion at the corner. The remainder of the expansion is produced by the reflected waves. To show this, the problems of interaction of a slip stream with a weak expansion wave and also the interaction of a shear layer with a wake expansion fan are solved. In reference 24, preliminary calculations from the characteristics program are presented.

The present paper presents a method which combines the rotational, axisymmetric characteristics with a viscous inner region, to determine near wake profiles. Imbedded shocks are considered in the characteristics solution. The surface boundary layer profiles at the separation point provide the initial conditions for the characteristics program in the supersonic region, while the subsonic part of the boundary layer is taken into account by dividing this portion of the boundary layer into strips and considering each strip to be governed by the one-dimensional flow equations including viscosity and thermal conductivity. The heat transfer and shear acting on each streamtube are computed from the average values of temperature and velocity in each strip. Details of the recirculating flow region are not considered in this analysis. The present analysis is useful to evaluate the flow field with reasonable accuracy to a few base diameters downstream of the body. At this location, the profiles could then be used as initial data to the available far wake analyses.

II. INVISCID ROTATIONAL FLOW FIELD

In order to analyze the near wake (cf. Fig. 1a) short of using the complete set of fluid mechanical equations available (Navier-Stokes), certain simplifying assumptions are made. For the present analysis they are:

1. A steady state solution is assumed.
2. No interaction from the subsonic part of the boundary layer (this alters the initial profiles and can be readily included if a more accurate determination of this effect is known.)
3. Expansion of the first streamline (Basic Streamline) takes place by means of a Prandtl-Meyer fan (A-B in Fig. 1b).
4. Basic Streamline (originally this streamline has a Mach number equal to M_1 at the trailing edge of cone) is a free streamline and its shape is determined by assuming a specified pressure distribution along it.

With the above assumptions, once all the variables are specified along a first family characteristic line emanating from the point of boundary layer separation on the cone, where the Mach number in the boundary layer $M = M_1 > 1.0$, the inviscid flow field may be analyzed by the method of rotational axisymmetric characteristics including imbedded shocks. The manner in which the pressure distribution along the basic streamline is specified will be described later.

As the flow near the shoulder expands, it will separate from the base of the cone and a Basic Streamline (B. S) is formed which will separate the inviscid outer flow from the viscous layer which is obtained by expanding the subsonic portion of the boundary layer. As the external streamlines

progress downstream, they will reach a point where their velocity will be in the direction of the axis. After this point is reached, the streamlines will again start curving outward and the compression waves formed by their divergence will coalesce into a trailing shock which may be weak or strong depending on the cone angle and flow conditions.

The details of the shoulder expansion region are better shown in Fig. (1b). Note that a lip shock may be formed by the concave curvature of the B. S. It is also noteworthy to mention that in order to solve for the entire supersonic region one would have to reflect the incoming expansion waves from the sonic surface. However, this is not done due to the complexity involved in the determination of the sonic surface which is imbedded in the viscous region, therefore, a streamline with an initial Mach number M_1 (Basic Streamline) is used as the reflection surface.

The problem is mathematically well-defined once all the conditions along the initial characteristic line, the point about which the Prandtl-Meyer expansion occurs, and the subsequent basic streamline, are specified. The analysis may then be subdivided into four separate unit problems:

1. Evaluation of an interior point (3) once values at (1) and (2), are known (see Fig. 2).
2. Reflection of a second family characteristic line from a pressure surface whose pressure is a given function of x. (Fig. 3)
3. Evaluation of conditions at a point which is obtained as a result of two characteristic lines of the same family intersecting (Fig. 4).
4. Extension of a shock wave once conditions at a point behind the shock are known (Fig. 5) (i.e., how to obtain C once A and B are known).

To solve the first unit problem the following equations are available:

along, $\frac{dY}{dX} = \tan(\theta + \mu)$ (first family characteristic)

$$\frac{\cot\mu}{V} dV - d\theta - \frac{J\sin\theta\sin\mu}{Y\cos(\theta+\mu)} dX + \frac{\cos\mu\sin^2\mu}{\gamma R\cos(\theta+\mu)} \frac{\partial S}{\partial N} dX - \frac{1}{V^2} \frac{\cos\mu}{\cos(\theta+\mu)} \frac{\partial H}{\partial N} dX = 0 \tag{1}$$

along, $\frac{dY}{dX} = \tan(\theta - \mu)$ (second family characteristic)

$$\frac{\cot\mu}{V} dV + d\theta - \frac{J\sin\theta\sin\mu}{Y\cos(\theta-\mu)} dX - \frac{\cos\mu\sin^2\mu}{\gamma R\cos(\theta-\mu)} \frac{\partial S}{\partial N} dx + \frac{1}{V^2} \frac{\cos\mu}{\cos(\theta-\mu)} \frac{\partial H}{\partial N} dX = 0 \tag{2}$$

along, $\frac{dY}{dX} = \tan\theta$ (streamline)

$$S = \text{const.} \tag{3}$$

$$H = \text{const.} \tag{4}$$

and also

$$\frac{V}{V_L} = \frac{M}{\sqrt{M^2 + \frac{2}{\gamma - 1}}}$$

All the variables may be nondimensionalized as follows:

$$h = \frac{H}{H_\infty} \quad s = \frac{S-S_\infty}{R} \quad v = \frac{V}{V_L} \quad x = \frac{X}{R} \quad y = \frac{Y}{R}$$

the differential equations are then reduced to a form amenable to solution by a computer, and they are

$$x_3 = \frac{(y_1 - y_2) + x_2 b_2 - x_1 g_1}{b_2 - g_1} \tag{5a}$$

$$y_3 = \frac{(x_2 - x_1) g_1 b_2 - y_2 g_1 + y_1 b_2}{b_2 - g_1} \tag{5b}$$

$$h_3 = h_2 - \frac{(h_2 - h_1) m_2 \Delta x_2}{n_1 \Delta X_1 + m_2 \Delta x_2} \tag{5c}$$

$$s_3 = s_2 - \frac{(s_2 - s_1) m_2 \Delta x_2}{n_1 \Delta x_1 + m_2 \Delta x_2} \tag{5d}$$

$$v_3 = \Big\{ A_1 v_1 + A_2 v_2 - (\theta_1 - \theta_2) + B_1 \Delta x_1 + d_2 \Delta x_2 + \frac{(s_2 - s_1)}{n_1 \Delta x_1 + m_2 \Delta x_2} \left[c_2 m_2 \Delta x_2 - c_1 n_1 \Delta x_1 \right]$$

$$+ \frac{(h_2 - h_1)}{2(n_1 \Delta x_1 + m_2 \Delta x_2)} \left[\frac{t_1 \Delta x_1}{h_1 v_1^2} - \frac{z_2 \Delta x_2}{h_2 v_2^2} \right] \Big\} \left[A_1 \sqrt{\frac{h_3}{h_2}} + A_2 \sqrt{\frac{h_3}{h_2}} \right]^{-1} \qquad (5e)$$

$$\theta_3 = \theta_2 - A_2 \left(v_3 \sqrt{\frac{h_3}{h_2}} - v_2 \right) + d_2 \Delta x_2 + (s_2 - s_1) \frac{c_1 m_2 \Delta x_2}{n_1 \Delta x_1 + m_2 \Delta x_2} - \frac{(h_2 - h_1)}{2 v_2^2 h_2 (n_1 \Delta x_1 + m_2 \Delta x_2)}$$

(5f)

All coefficients are defined in the List of Symbols.

The equations for the reflection of an expansion line from a pressure surface are derived by assuming that the B. S. passes through a point (1) (see Fig. 3) and conditions at a point off the streamline (2) are known; the continuation of the streamline is desired and this is done by locating point (3).

A second family characteristic line from 2 to 3 and a streamline from 1 to 3 are used. The equations available are then:

$$x_3 = \frac{y_2 - y_1 + x_1 e_1 - x_2 b_2}{e_1 - b_2} \qquad (6a)$$

$$y_3 = \frac{y_2 e_1 - y_1 b_2 - e_1 b_2 (x_2 - x_1)}{e_1 - b_2} \qquad (6b)$$

$$s_3 = s_1 \qquad (6c)$$

$$h_3 = h_1 \qquad (6d)$$

$$M_3^2 = \frac{2}{\gamma - 1} \left\{ \left(1 + \frac{\gamma - 1}{2} M_1^2\right) \left(\frac{p_1}{p_3(x)} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right\} \qquad (6e)$$

$$v_3 = \frac{M_3}{\sqrt{M_3^2 + \frac{2}{\gamma - 1}}}$$

$$\theta_3 = \theta_2 + A_2 \left(v_2 - v_3 \sqrt{\frac{h_3}{h_2}} \right) + d_2 \Delta x_2 - c_2 (s_3 - s_2) - \frac{A_2}{v_2} \left(\frac{h_3 - h_2}{h_1 + h_2} \right) \qquad (6f)$$

where all the functions are evaluated in the above order.

In the case of two intersecting characteristic lines of the same family (AC and BD in Fig. 4) a shock is assumed to form at the intersection point E. By using the values at E_-(on line AC) and E_+(on line BD), the strength of the shock is found. Since it turns out that the shock is a very weak one (i.e., $\frac{\Delta s}{s} \leq 2$-3%), the line BD is extended by using the first family characteristic line in front of AC as the line from which a second family is eminated.

III. SHEAR LAYER

The portion of the boundary layer whose Mach number is subsonic before the expansion at the trailing edge cannot be analyzed by the method of characteristics since the shear and heat conduction effects may be large in this region; in addition, a portion of this layer remains subsonic even after expansion at the corner. It is therefore analyzed by conserving mass momentum, and energy in individual stream tubes wherein the flow is assumed to be one dimensional, including the effects of shear and heat conduction between different stream tubes. This procedure also determines the pressure distribution along the outer most stream tube which is adjacent to the characteristic field. By matching the two regions, therefore, the analysis can proceed downstream in a consistent manner.

The subsonic part of the boundary layer is divided into "n" strips and it is assumed that each strip expands inviscidly and adiabatically from p_e to p_b as if it were one dimensional. As soon as this corner expansion is completed, each streamtube is followed by using the one dimensional equations with shear and heat transfer as given, for example, by Shapiro[25] (see Fig. 6). The energy, momentum, and continuity equations in nondimensional form are

$$\frac{dh_i}{dx} = 2\pi \frac{M_e}{M_i} \frac{p_e}{A_i p_i} \left[q_i y_i - q_{i-1} y_{i-1}\right] \tag{7}$$

$$\frac{dM_i^2}{dx} = -2M_i^2 \left(1 + \frac{\gamma-1}{2} M_i^2\right) \left[\frac{1.0}{\gamma p_i M_i^2} \left(\frac{dp_i}{dx}\right) \pm \frac{1}{2.0 h_i} \frac{dh_i}{dx} + \frac{c_{f_{T_i}}}{y_i - y_{i-1}}\right] \tag{8}$$

$$\frac{dA_i}{dx} = A_i \left\{ \frac{(1-M_i^2)}{\gamma p_i M_i^2} \frac{dp_i}{dx} + \left(1 + \frac{\gamma-1}{2} M_i^2\right) \frac{1}{h_i} \frac{dh_i}{dx} + \frac{\left(1 + (\gamma-1) M_i^2\right) c_{f_{T_i}}}{y_i - y_{i-1}} \right\} \tag{9}$$

where $$Q_i = K \frac{\partial T}{\partial y} \doteq K_i \frac{(T_i - T_{i-1})}{(y_i - y_{i-1})}$$

$$c_{f_{T_i}} \equiv c_{f_i} - c_{f_{i+1}}$$

$$\rho \frac{V^2}{2} c_{f_i} = \mu \frac{\partial V}{\partial Y} = \mu_i \frac{\Delta V_i}{\Delta Y_i} = \mu_i \left(\frac{V_i - V_{i-1}}{Y_i - Y_{i-1}} \right)$$

The unknowns in the above equations are $\bar{A}_i, M_i, h_i$. The pressure $p_i(x)$ is assumed to be known and is equal to the value as given by the inviscid characteristic program.

Zero heat transfer is assumed along the basic streamline and a mild stagnation temperature variation is assumed in the recirculation region. Internal diffusion takes place by virtue of heat transfer and shear across the strips. Since, for a given pressure distribution $p(x)$, the basic streamline has been obtained from the characteristic program, this line is used as a reference line from which the radial dimension of the n strips is measured when the "correct" pressure distribution is used along the basic streamline. The boundary line of the inner most pressure distribution used to determine the characteristic field and the location of the basic streamline is therefore iterated on to obtain this condition before the analysis proceeds to the next streamwise calculation of the matching flow fields. In this manner, the dividing streamline and the rear stagnation region can be obtained once the base pressure p_b is specified.

IV. RESULTS

Few detailed experimental profiles of flow parameters in the near wake of blunt based cones are available in the literature. Most experimental papers dealing with this subject present results in terms of variables which do not describe the details of the flow field. For example, there is a significant amount of information available on base pressure, heat transfer to base or rear stagnation point location while detailed measurements of the flow field are not presented.

Two recent papers which present local profiles of pressure, temperature, etc., at various downstream stations are by Schmidt and Cresci[26] and Bauer[27]. The free stream Mach number, Reynolds numbers and other pertinent test conditions are presented below for the two experiments.

Table (1) Experimental Test Conditions

M_∞	θ_w	D	Pt_∞	Tt_∞	Tw	$\frac{P_b}{P_\infty}$	X	Re_∞	Ref.
8.0	10^o	8"	100psi	1700^oR	544^oR	0.40	3.0	2×15^5	19
3.0	12.5	1"	8.26psi	532^oR	532^oR	0.24	2.1	13×10^5	20

The above conditions were used as inputs for the characteristics program. In both cases, the experimental values of the base pressure were used in conjunction with the axisymmetric, rotational program. Since this information can be obtained from empirical correlations (cf. Ref. 28, for example) for different flow conditions, no generality is lost by this assumption. Once the base pressure is known, the pressure distribution from the base to the rear stagnation point can be determined if the maximum Mach number (minimum pressure) is specified. As explained in the previous sections, the location of the rear stagnation region is obtained by matching

the viscous shear layer with the inviscid characteristic program. The viscous shear layer from the cone base to the rear stagnation region was found to be governed principally by the effect of heat transfer from the recirculation region while downstream of the stagnation region, internal shear produced the largest effect. The temperature of the layer adjacent to the shear layer was assumed to be either (i) a constant or (ii) vary from the cone surface temperature at x = 0 to the recirculation region temperature at the rear stagnation point. As seen from Fig. (9-a) the effect due to this variation appears to be negligible. For the Mach eight conditions, profiles at different x stations are obtained and values of pitot pressures, static pressure and stagnation enthalpy are plotted and compared with the experimental results in Figures (7) through (9). The stagnation enthalpy profiles are seen to agree very well, especially in the region close to the rear stagnation point. The accuracy of the analysis decreases in the downstream direction as the region dominated by diffusive effects grows into the flow field computed by characteristics, thereby invalidating the basic assumption of an inviscid outer flow. The pitot pressure profiles are seen to be in good agreement up to $x/D = 3.25$. In contrast to the other two sets of profiles, the static pressure profiles are less accurate close to the rear stagnation point. This is believed to be due to two effects. First, it is much more difficult to accurately measure static pressure in the recirculation and stagnation region due to probe interference, and second, the theory is able to determine the local pressure distribution at every point downstream of rear stagnation point in a self-consistent manner, while in the rear stagnation region the uniqueness of the static pressure distribution is not guaranteed.

It is also seen that at each x station, the location of the lip and recompression shock can be predicted with good accuracy. In Fig. (10) one sees the flow field for this case. The light lines indicate the various first family characteristics emanating from the basic streamline (for clarity, second family lines are omitted), while the two darker lines show the location and shape of the lip and recompression shock.

In Fig. (11) pitot pressure profiles for the Mach 3.0 case are presented. Due to the few results published by Bauer, the only variables which were compared were the pitot pressure profiles; again comparison between theory and experiment is reasonably good and the location of the lip shock is predicted within experimental accuracy.

One may note that the Crocco-Lees type of singular behavior doesn't appear in this analysis since the critical region is not analyzed in detail.

V. CONCLUDING REMARKS

The present analysis of the near wake is able to predict (with reasonable accuracy) flow conditions and shock shapes at different locations without the necessity of analyzing the recirculation region in detail. The advantage of this is immediately evident in that the problem of the recirculation region is formidable due to the complexity of the differential equations which govern the flow and the specification of boundary conditions along an undetermined boundary. It may therefore be inferred that unless one is interested in the recirculation region per se, the detailed solution to this region will not play an extremely important role in the downstream flow. The complete solution of the recirculation region has been replaced by the specification of several conditions in this region: 1) the base pressure (ref. 23), 2) an average temperature of the recirculation region (ref. 24), 3) maximum Mach number (minimum pressure) along the centerline (ref. 25). Since these parameters empirically have been correlated under different flow conditions in the referenced papers, these conditions can be readily obtained.

The following conclusions may be drawn: 1) stagnation enthalpy profiles are relatively insensitive to the shape of the initial profiles and to the heat transfer from the recirculation region, 2) while this is also true for the inviscid portion of the stagnation and static pressure profiles, the shear layer is quite sensitive to these conditions.

As a result, further refinements and/or extensions to the present work should deal with 1) detailed analysis of the initial expansion of a compressible shear layer, 2) analysis of the rear stagnation region, and

3) a better representation of the heat transfer and shear between the shear layer and the recirculation region.

The freedom in inputs of the present theory will allow for calculations of near wake of more general shaped bodies (spheres or blunt bodies). The only modification involved would be the alteration of inputs along the characteristic line emanating from the separation point.

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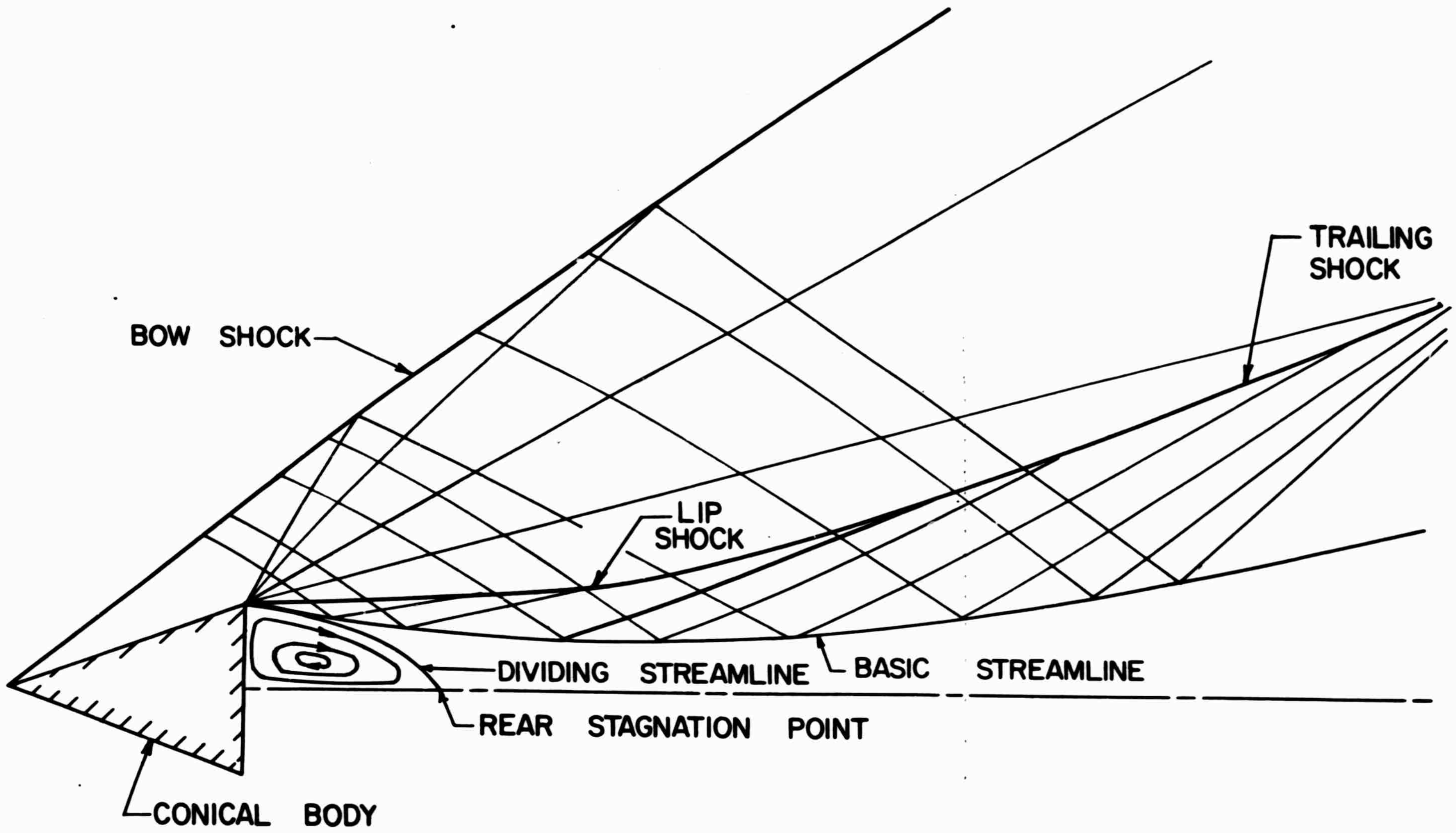
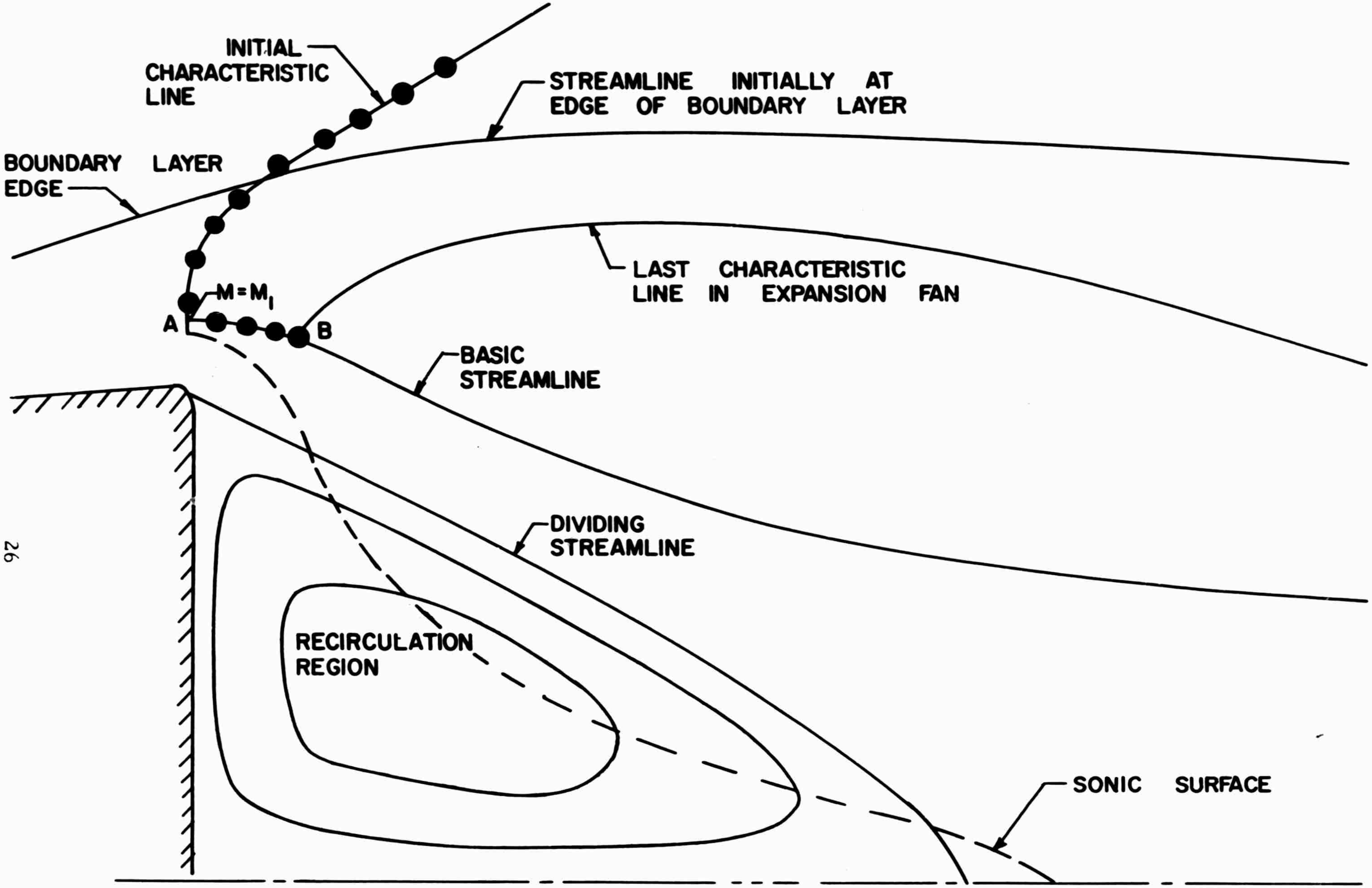


(a) GENERAL DESCRIPTION OF NEAR WAKE

FIG. (I) SCHEMATIC OF FLOW FIELD



(b) DETAILS OF SEPARATION REGION

FIG. (1) SCHEMATIC OF FLOW FIELD

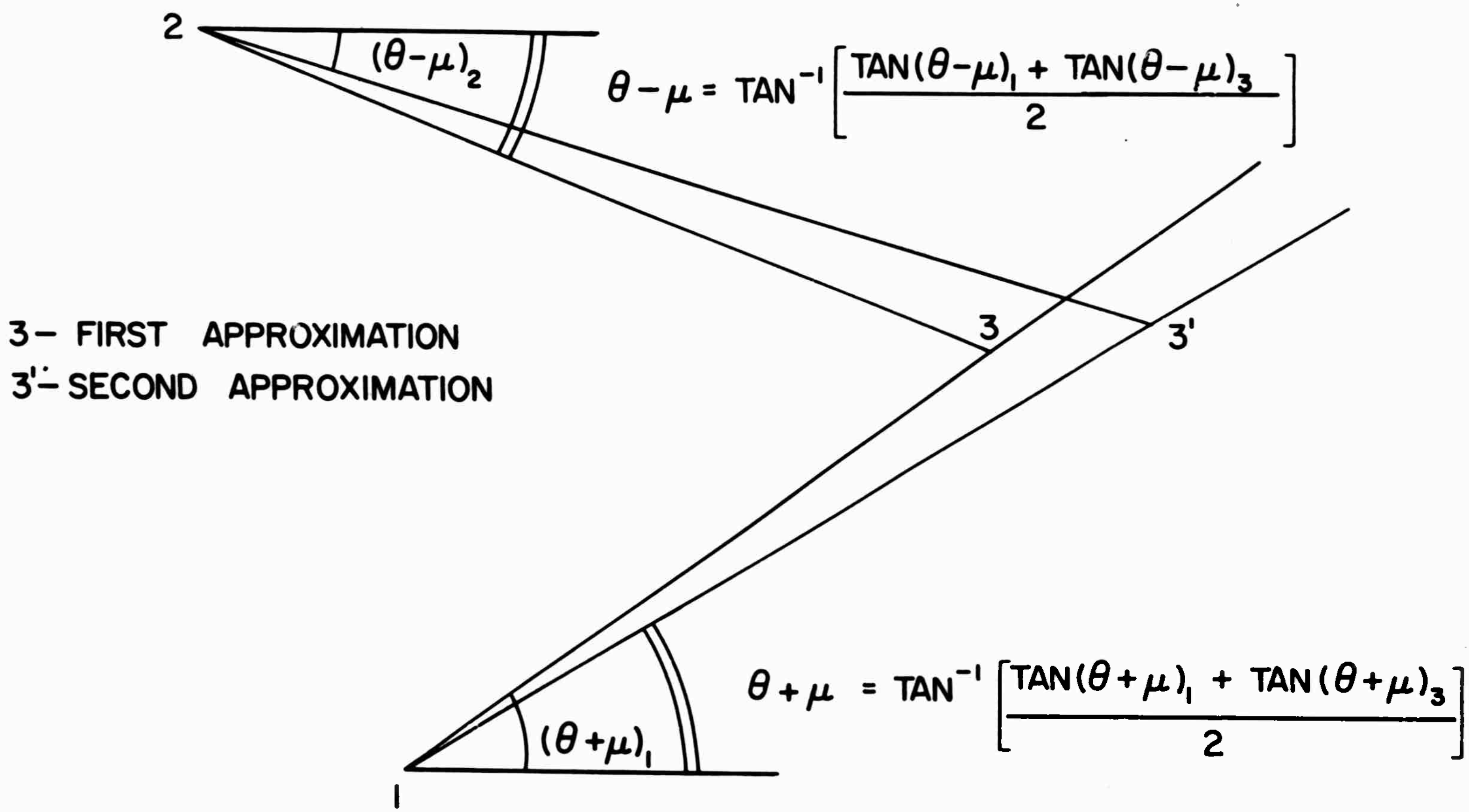


FIG. (2) PROCEDURE USED TO EXTEND CHARACTERISTIC LINE

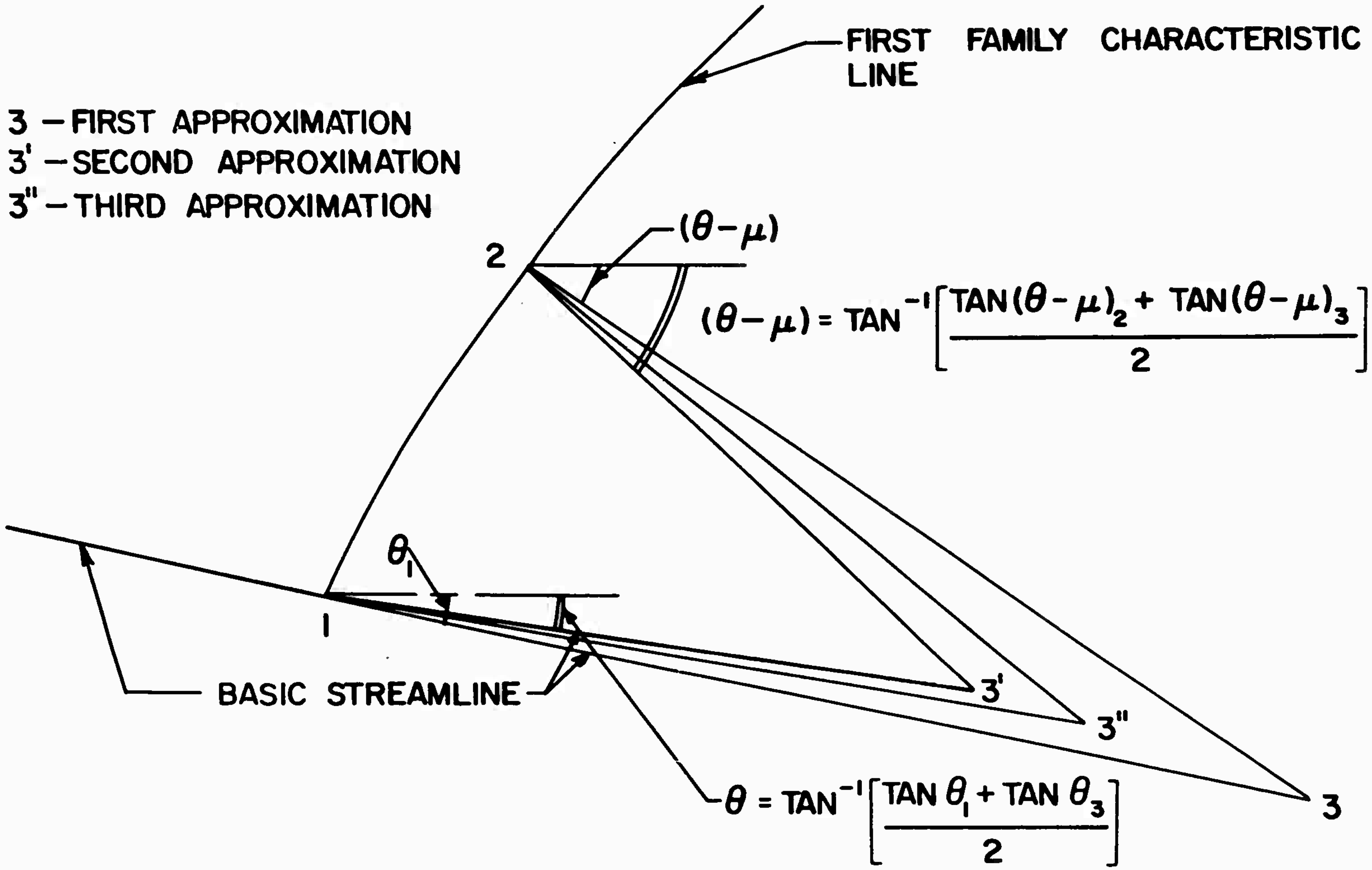


FIG. (3) PROCEDURE USED TO EXTEND SHAPE OF STREAMLINE WHOSE INITIAL MACH NUMBER IS M_1

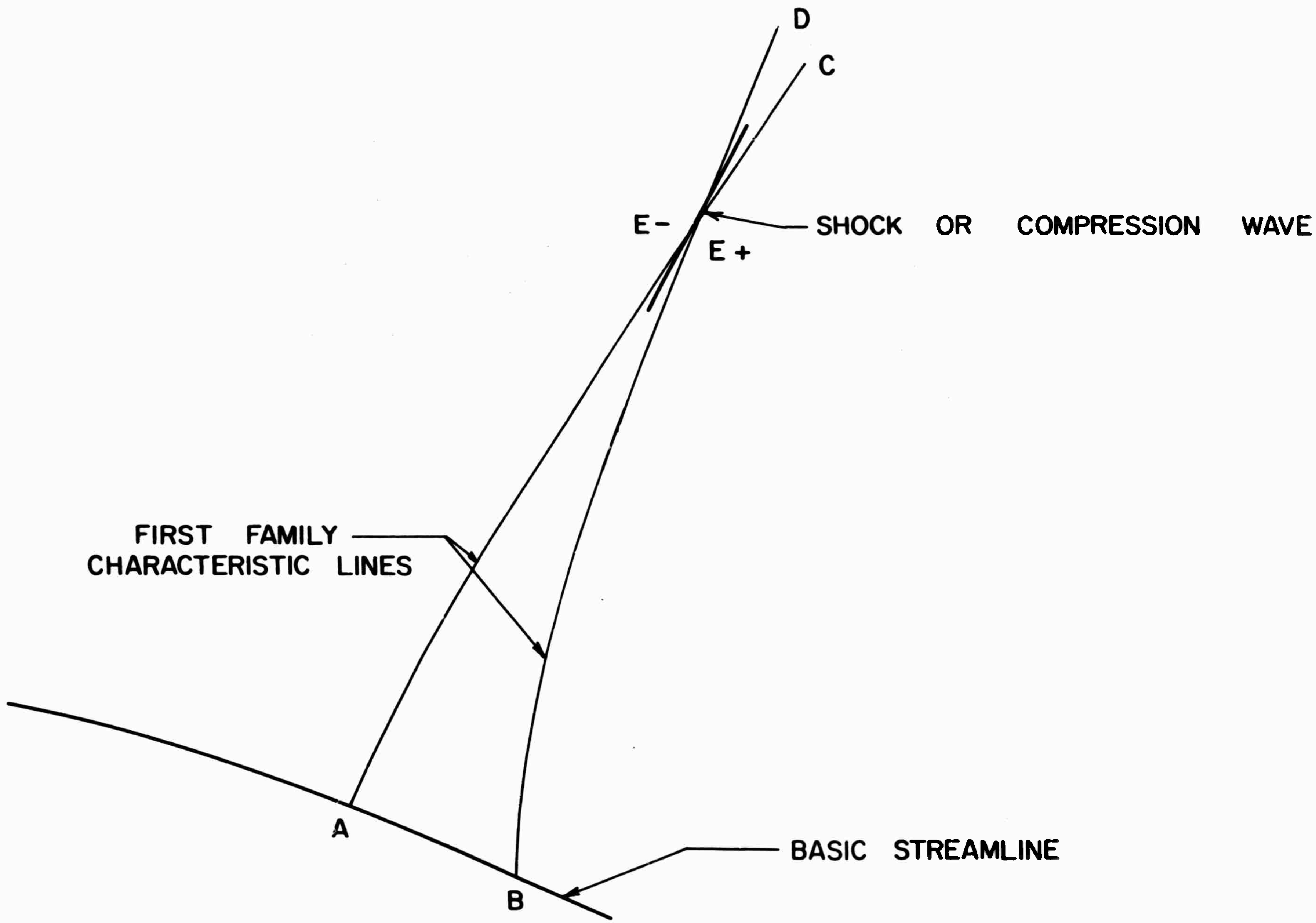


FIG. (4) INTERSECTION OF TWO FIRST FAMILY CHARACTERISTIC LINES

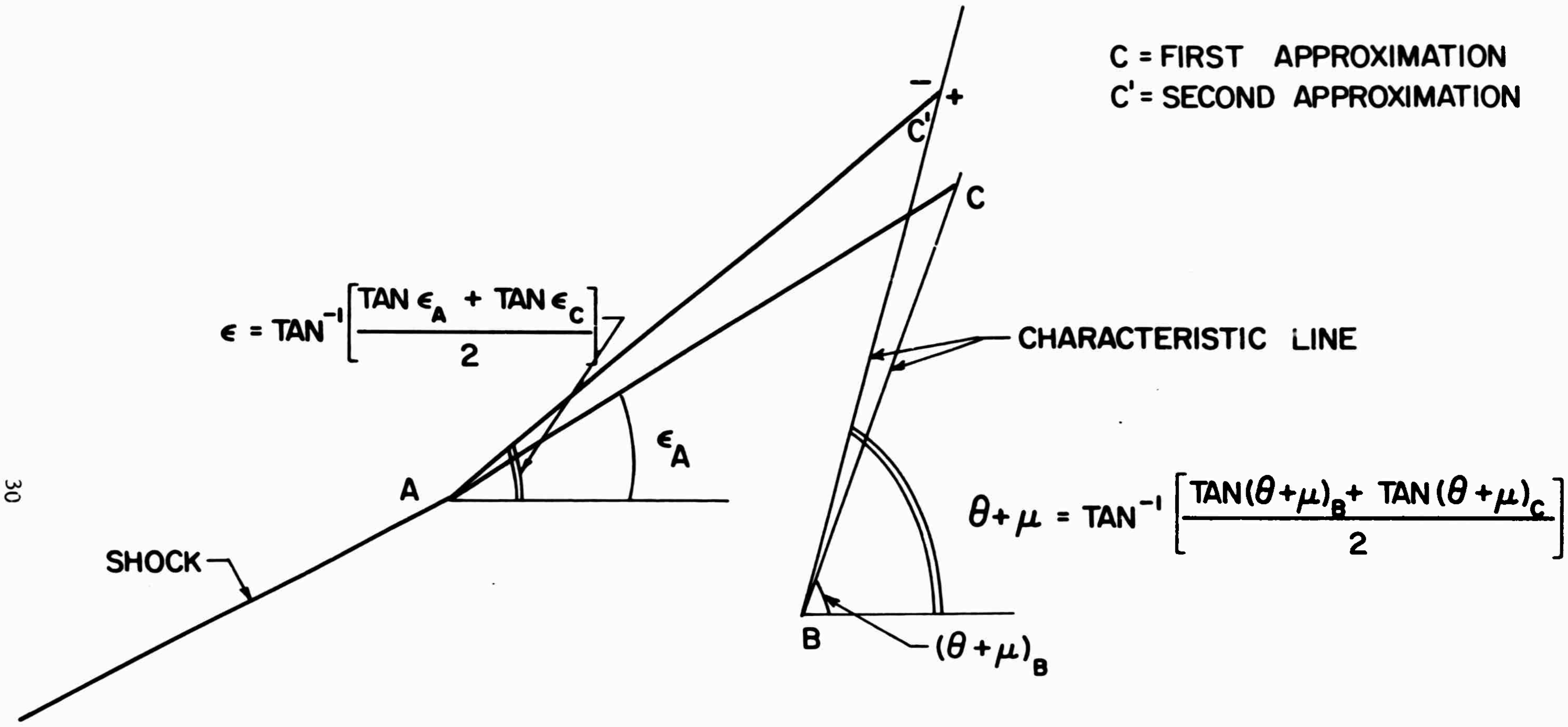
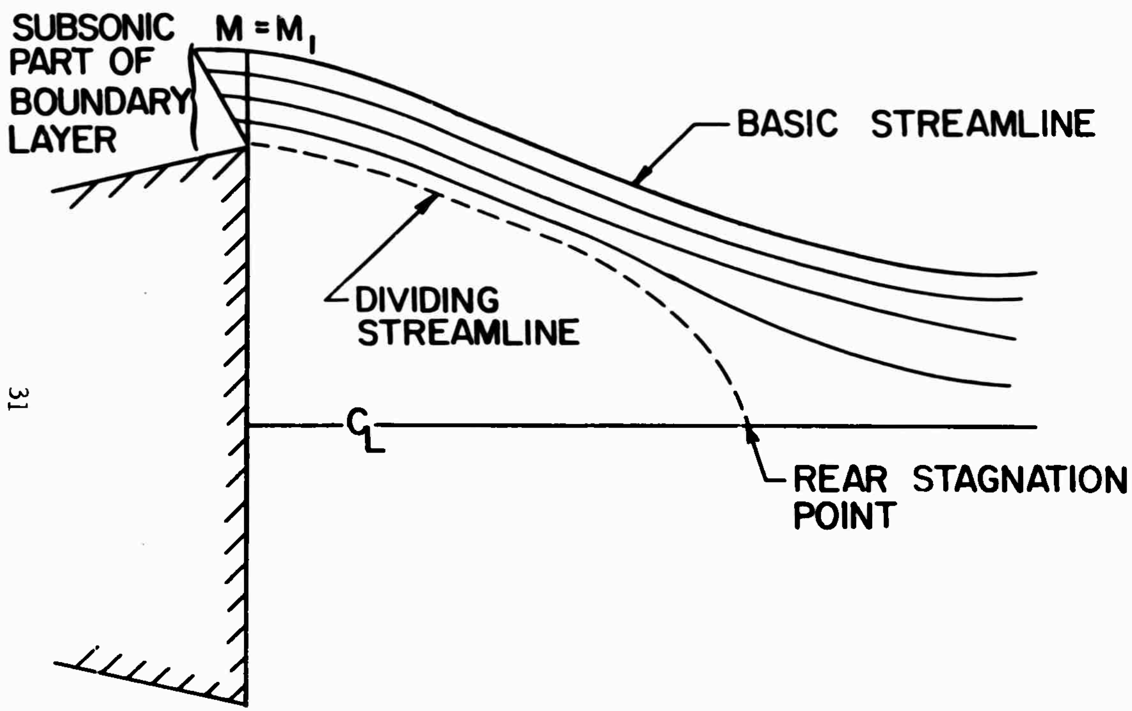
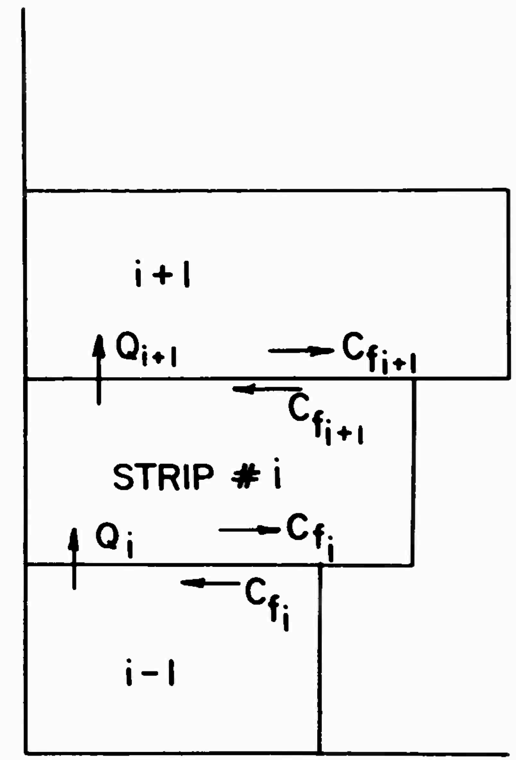


FIG. (5) PROCEDURE USED TO EXTEND SHAPE OF SHOCK



(a) GENERAL DESCRIPTION



(b) MECHANISM FOR MOMENTUM AND ENERGY DIFFUSION FOR STEP PROFILES

FIG. (6) SCHEMATIC OF SHEAR LAYER ANALYSIS

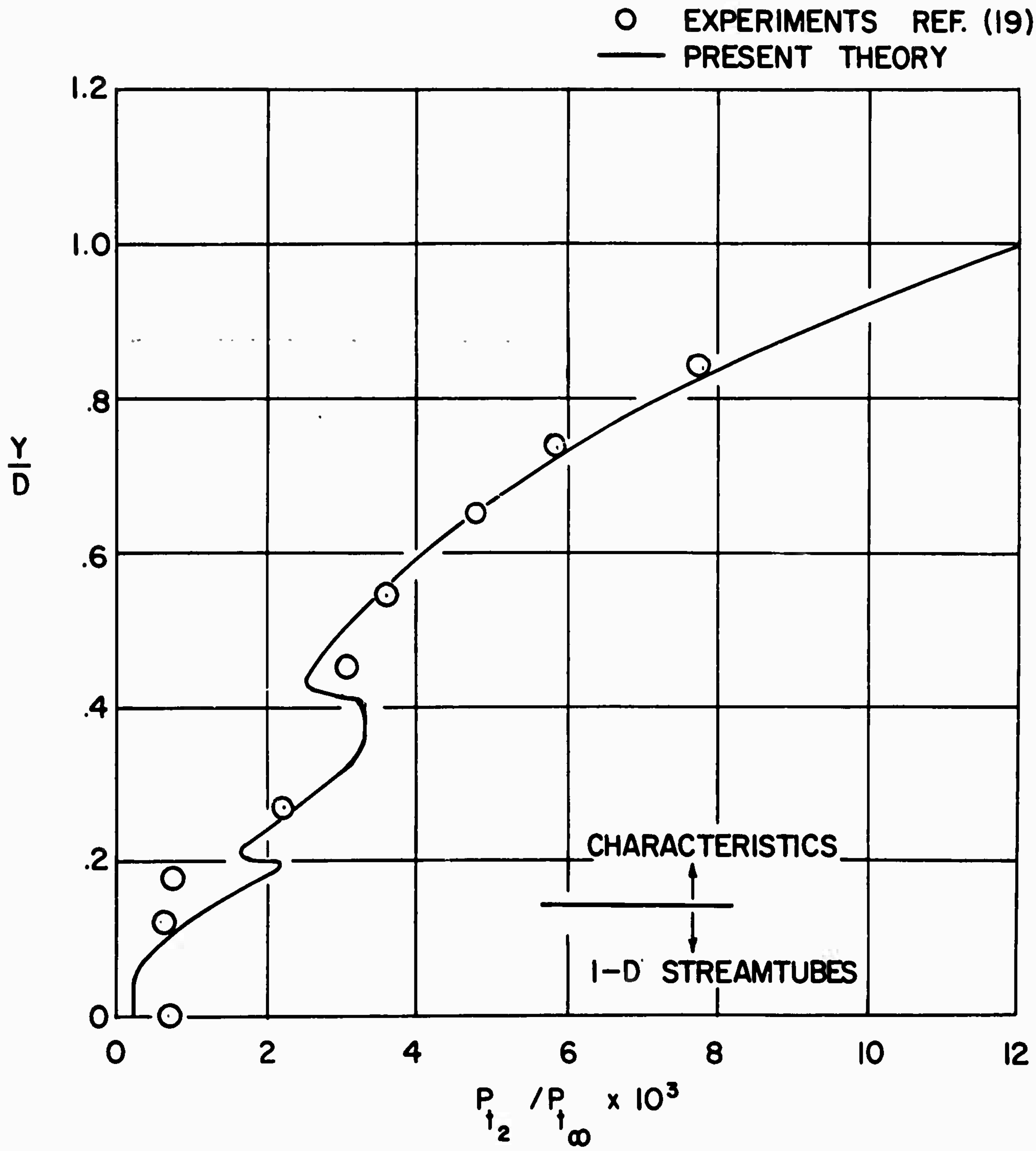


FIG. (7) PITOT PRESSURE PROFILE, $M_\infty = 8.0$

(a) $\frac{X}{D} = 2.14$

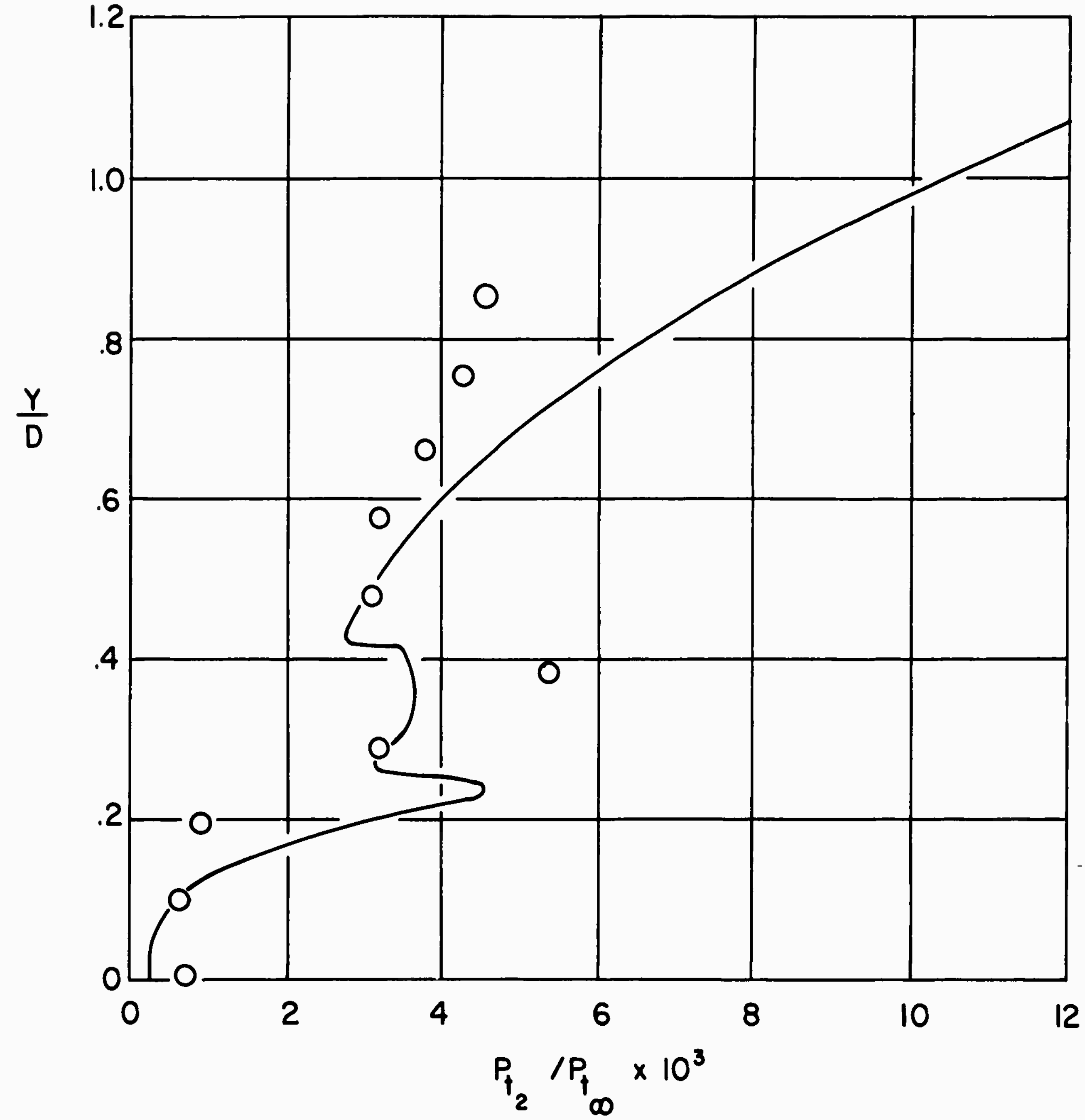


FIG. (7) PITOT PRESSURE PROFILE, $M_{\infty}=8.0$

(b) $\frac{X}{D} = 2.50$

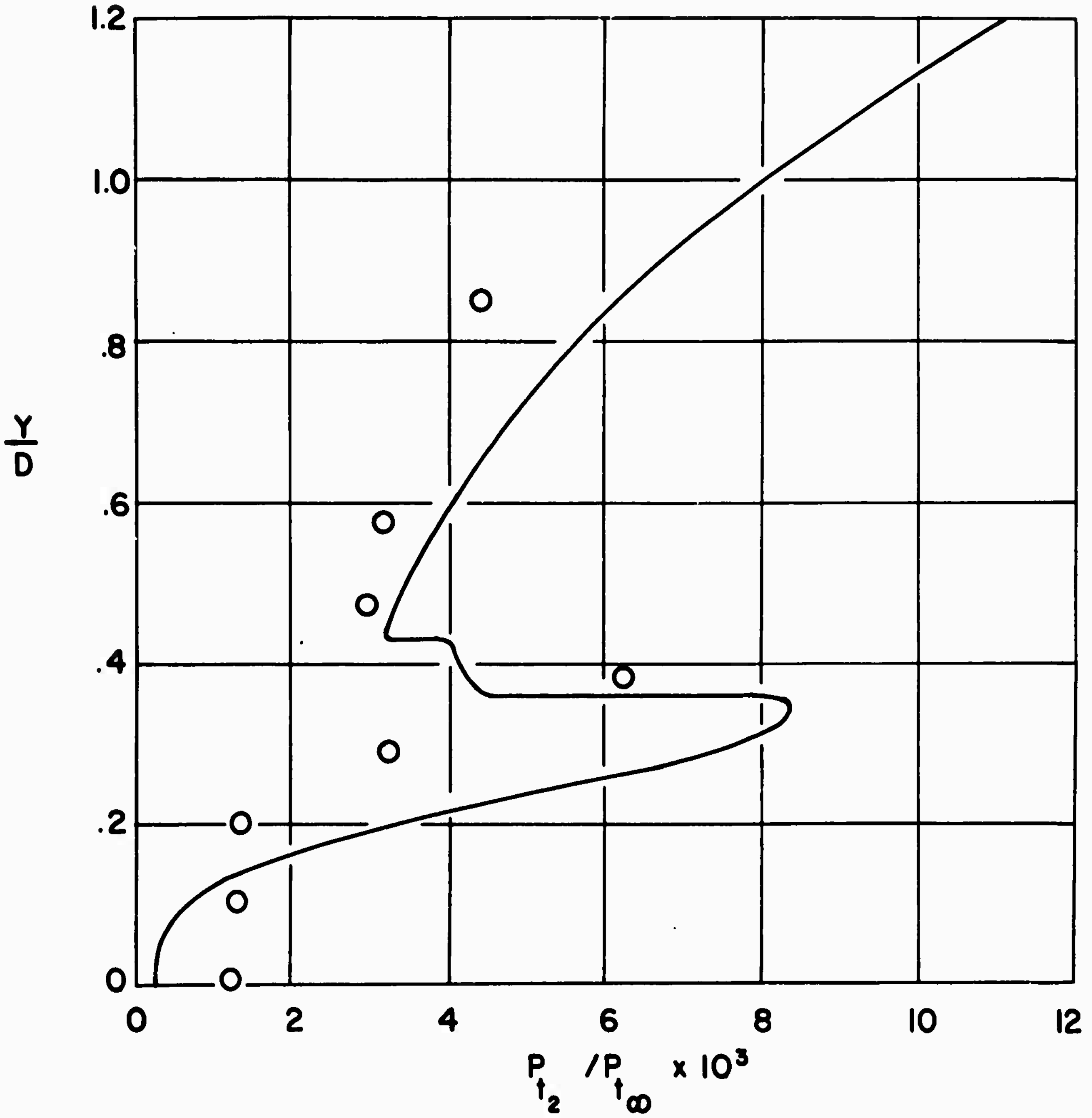


FIG. (7) PITOT PRESSURE PROFILE, $M_\infty = 8.0$

(c) $\frac{X}{D} = 3.25$

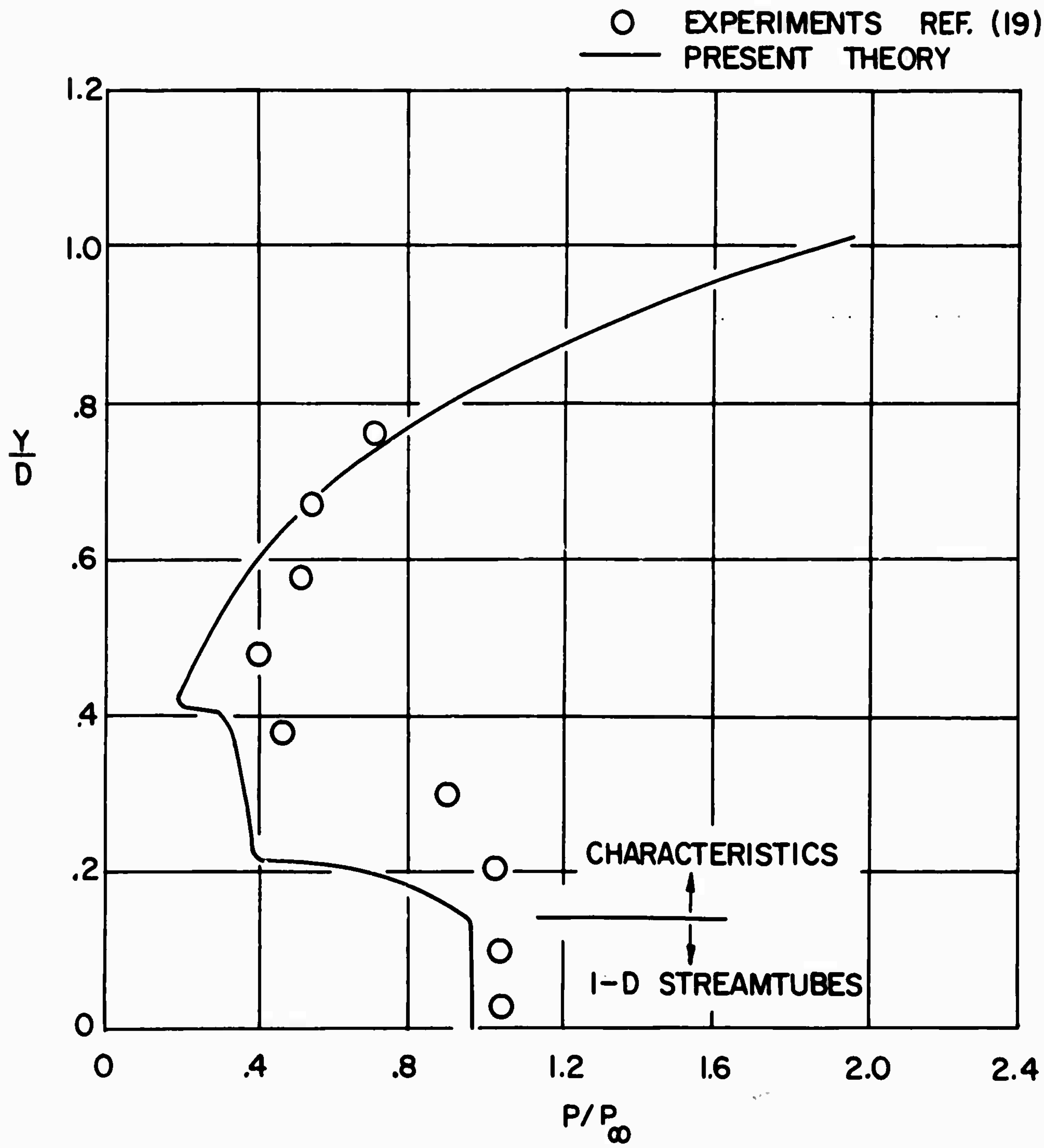


FIG. (8) STATIC PRESSURE PROFILE, M = 8.0

(a) $\frac{X}{D}$ = 2.14

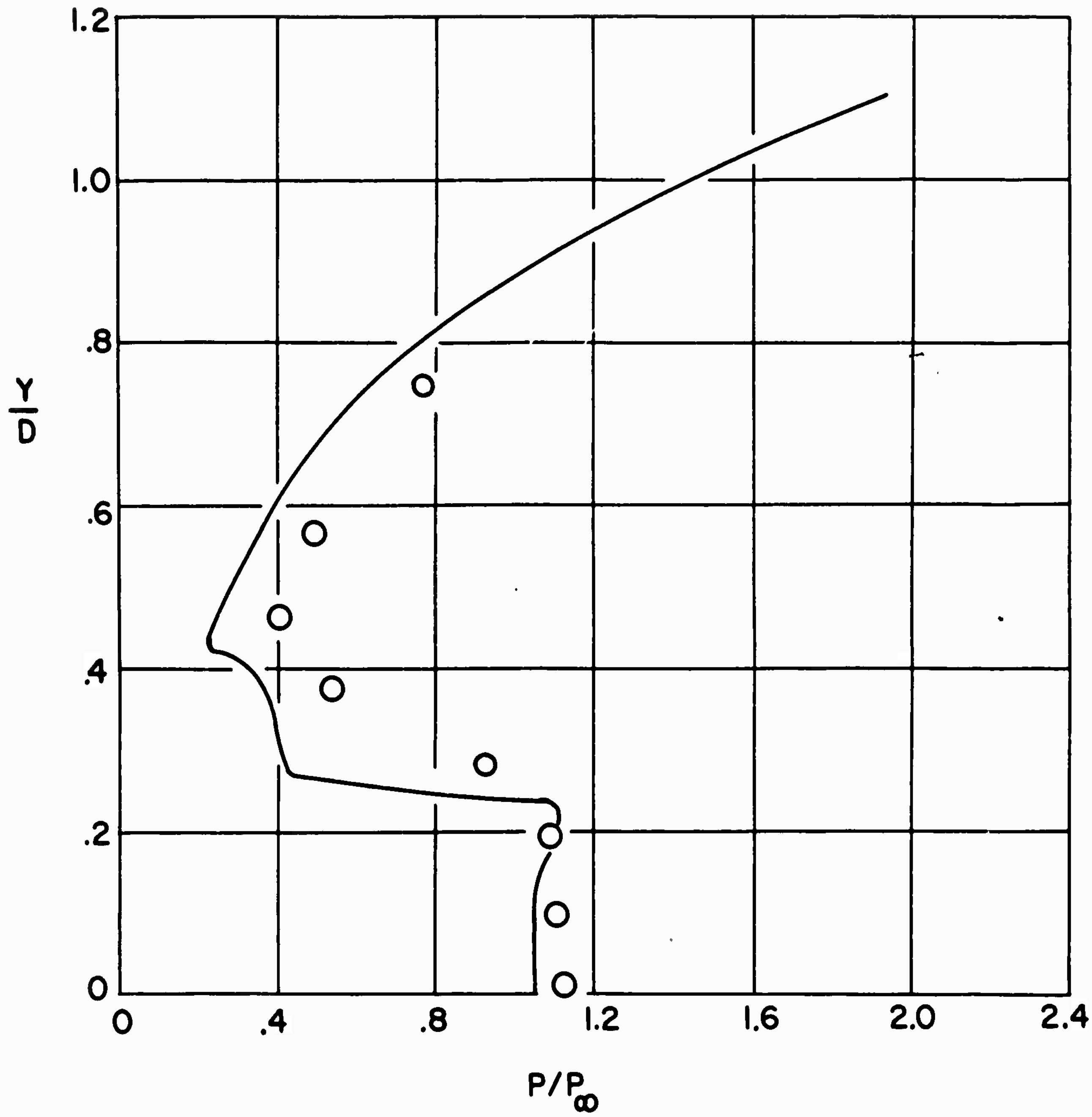


FIG. (8) STATIC PRESSURE PROFILE, $M_\infty = 8.0$

(b) $\frac{X}{D} = 2.50$

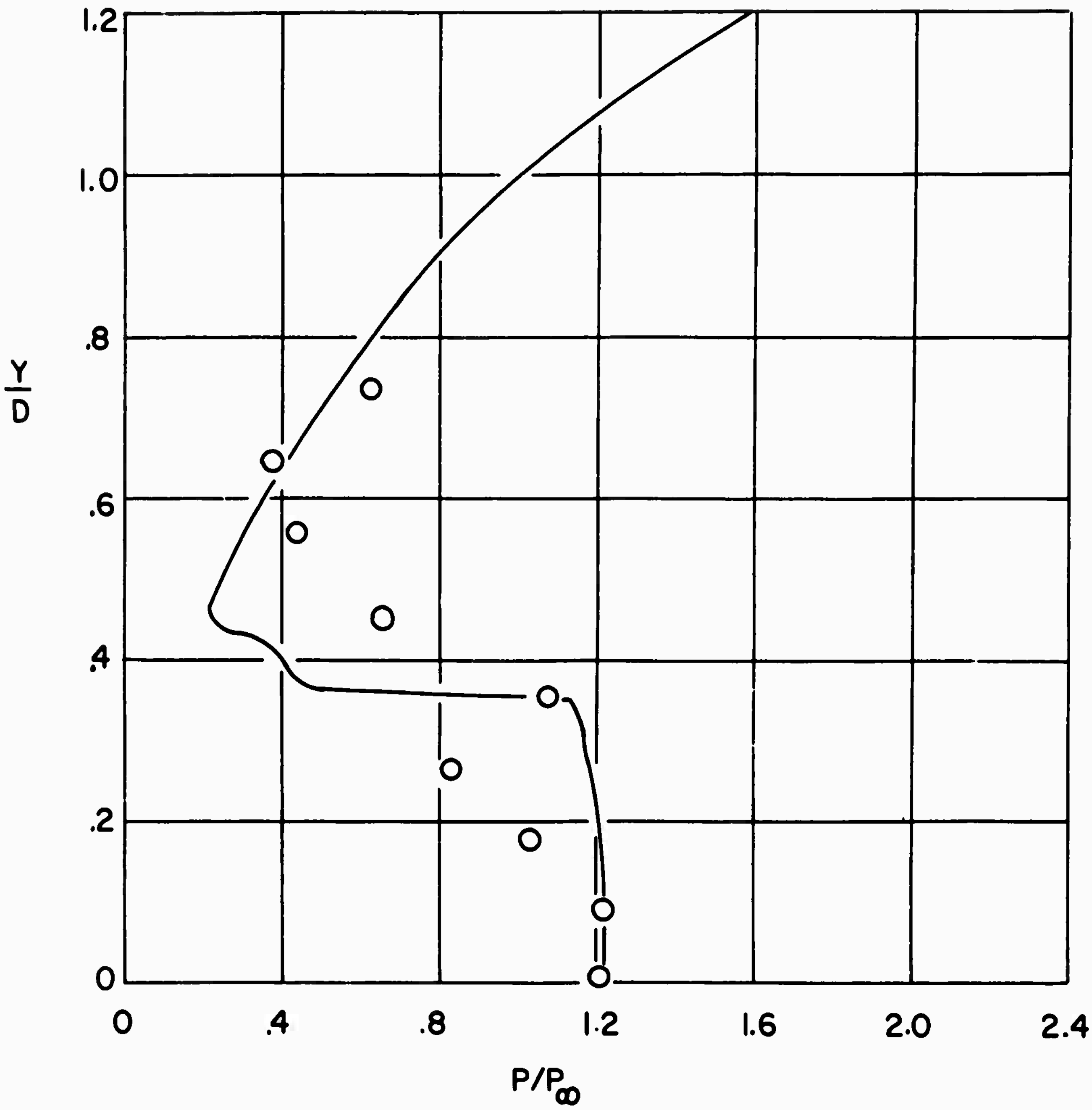


FIG. (8) STATIC PRESSURE PROFILE, $M_{\infty} = 8.0$

(c) $\frac{X}{D} = 3.25$

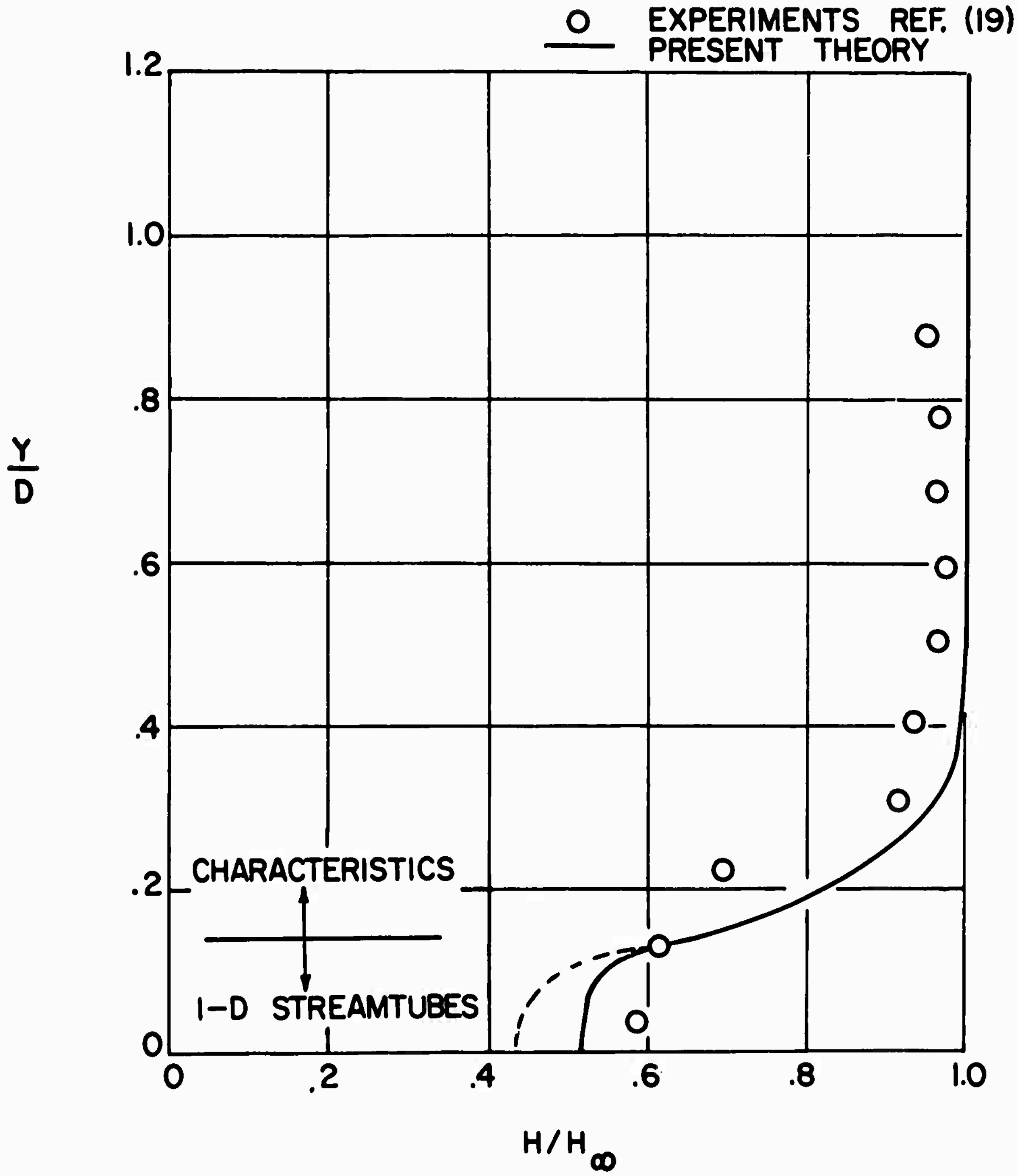


FIG. (9) STAGNATION ENTHALPY PROFILE, $M_\infty = 8.0$

(a) $\frac{X}{D} = 2.14$

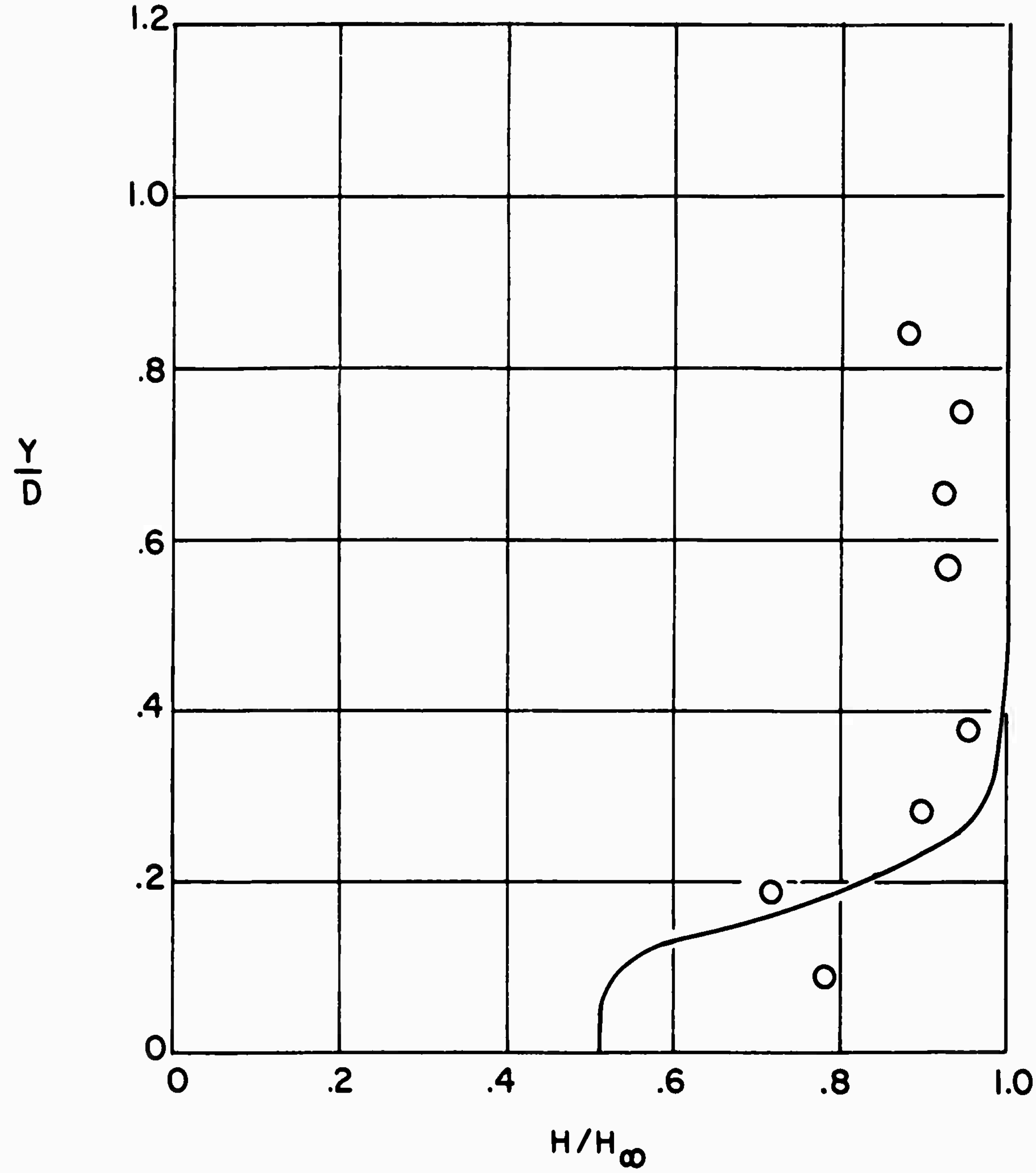


FIG. (9) STAGNATION ENTHALPY PROFILE, $M_\infty = 8.0$

(b) $\frac{X}{D} = 2.50$

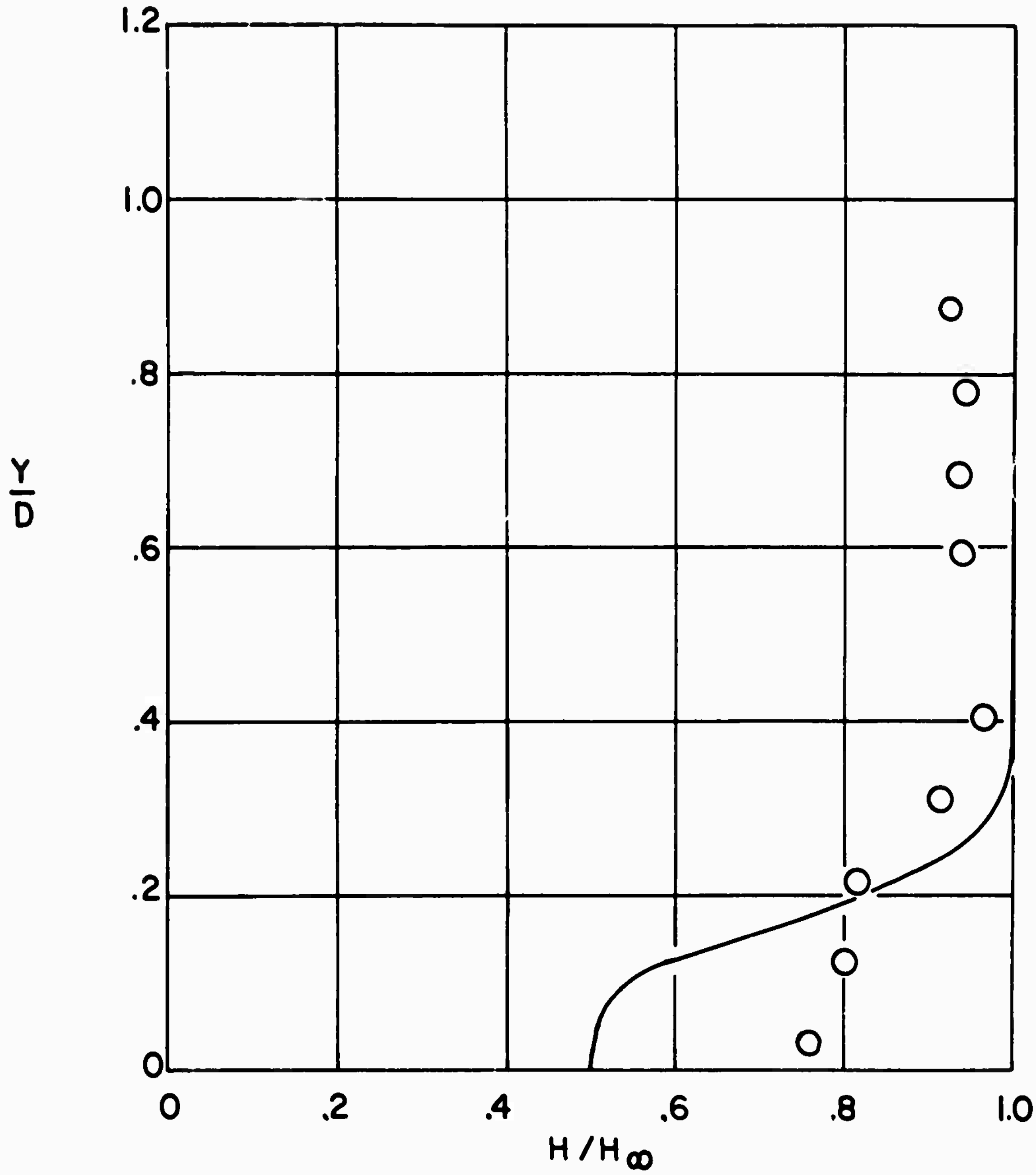


FIG. (9) STAGNATION ENTHALPY PROFILE,
$M_{\infty} = 8.0$
(c) $\frac{X}{D} = 3.25$

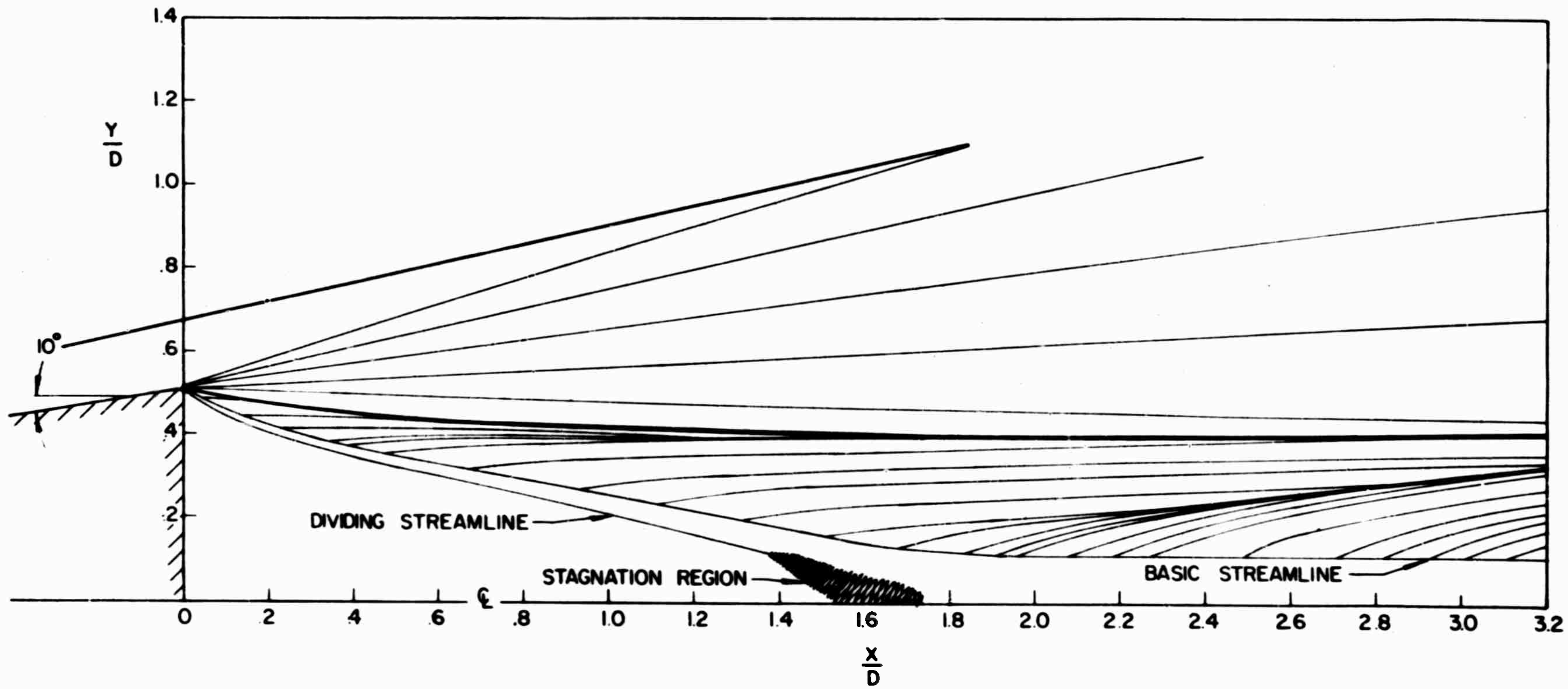


FIG. (10) FLOW FIELD $M_{\infty}=8.0$, $T_{s_{\infty}} = 1750°R$, $P_{s_{\infty}} = 100$ psia

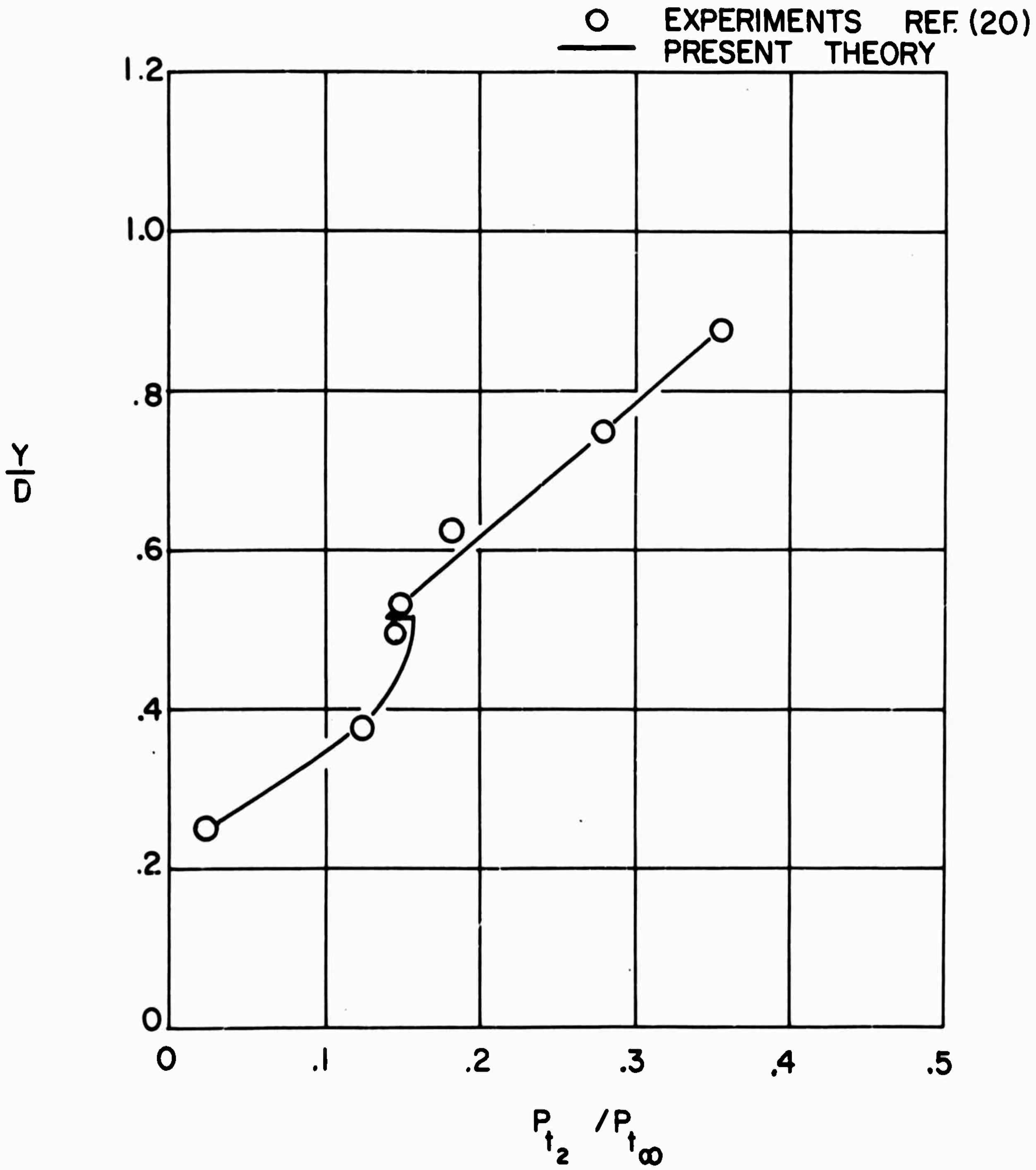


FIG. (11) PITOT PRESSURE PROFILE, $M_\infty = 3.0$.
(a) $\frac{X}{D} = 0.75$

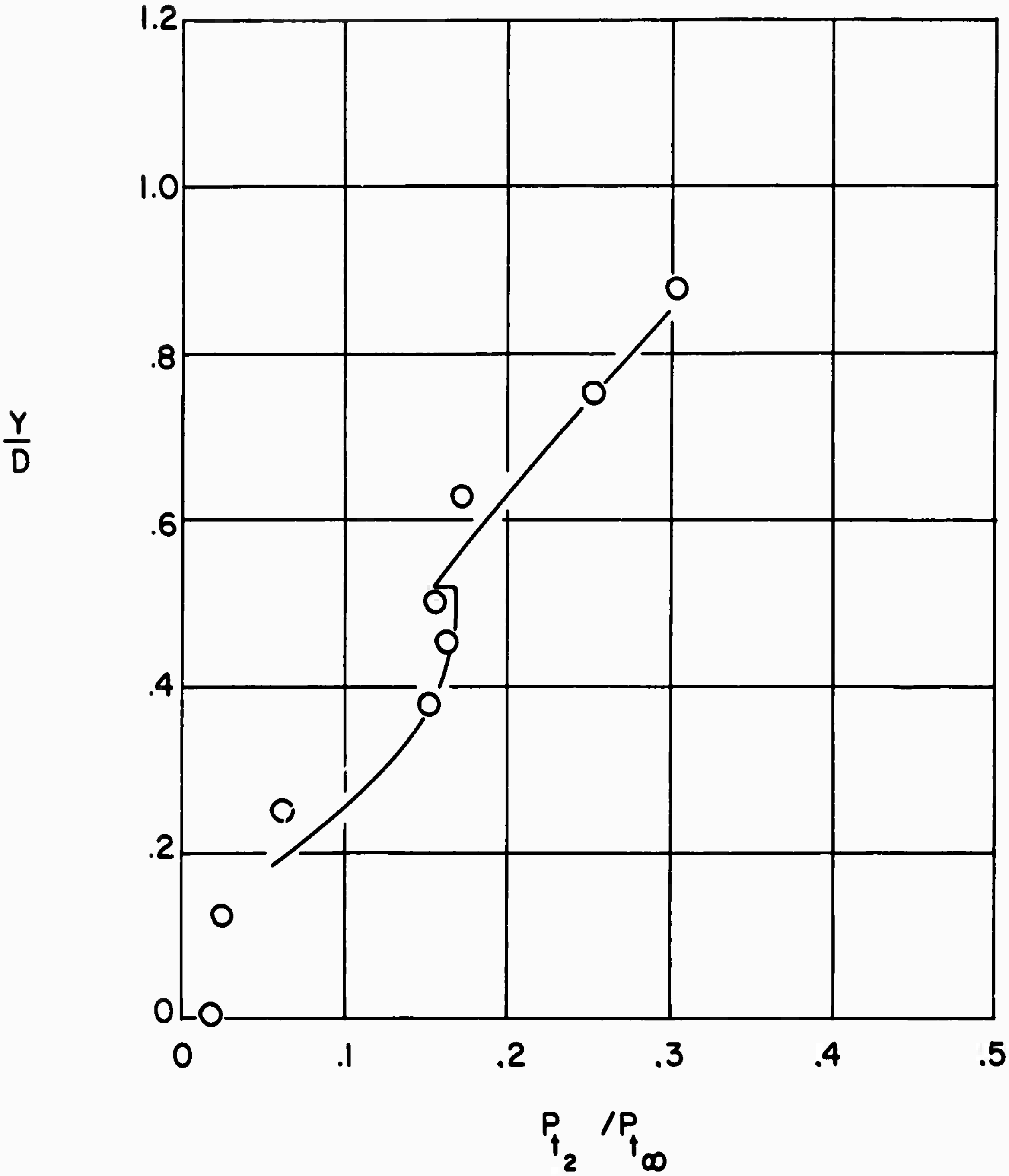


FIG. (11) PITOT PRESSURE PROFILE, $M_\infty = 3.0$

(b) $\frac{X}{D} = 1.0$

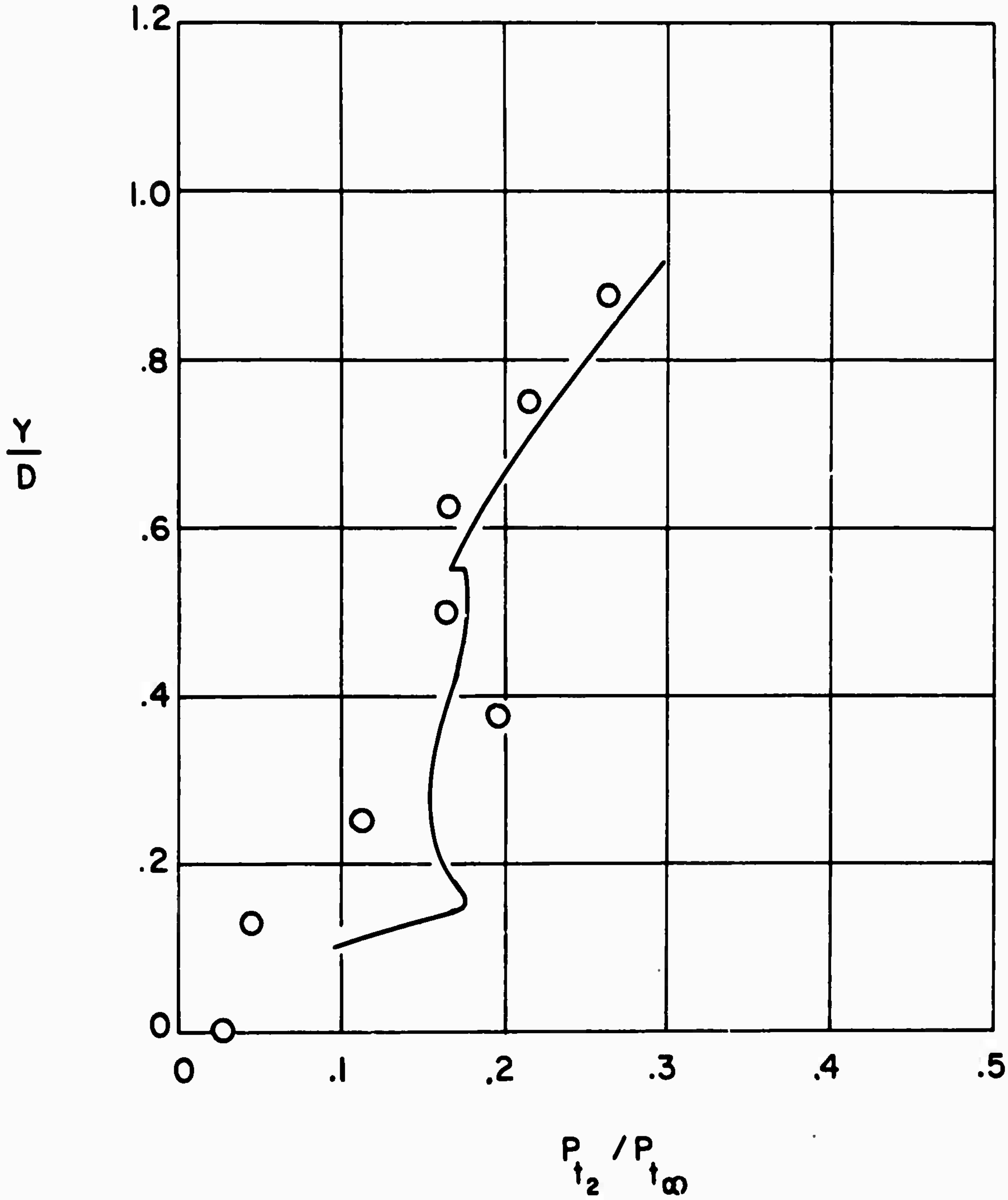


FIG. (11) PITOT PRESSURE PROFILE, $M_\infty = 3.0$

(c) $\frac{X}{D} = 1.25$

APPENDIX A

DESCRIPTION OF CHARACTERISTIC PROGRAM

The program is capable of analyzing the trailing edge expansion and the near wake for any given cone angle, Mach number, and free stream stagnation conditions. It consists of 13 different sections; eight of these are function subroutines for the different coefficients, the remaining five are MAIN, CINPUT (Calculate INPUT), CHAR (CHARacteristic), CSHOCK, (Cone SHOCK), DIVST (DIViding STreamline), subroutines. A flow chart for each of these is provided by Figs. A-1 to A-5.

At the start, the MAIN program directs the computer to the CINPUT subroutine. The function of this subroutine is to evaluate and store in the memory of the computer the initial characteristic line and the conditions along a streamline which originates from a point where the Mach number is equal to M_1. The conditions along this streamline change by means of a Prandtl-Meyer expansion procedure. In order to evaluate the initial first family characteristic line, the "viscous" part of the line is solved and matched with the "inviscid" region. The inviscid or potential characteristic line is read in to the computer from Sims' tables[22]. This line is then shifted a little upstream and/or downstream until the two lines match to the desired degree of accuracy. To evaluate the viscous part of the characteristic line, the only inputs which are necessary are various values of Mach number varying from M_1 to M_e. Once the Mach number is known, the velocity u/u_e can be found (all the values at the edge of the boundary layer are assumed to be equivalent to the inviscid values on the cone and are obtained from Sims' tables). Once u/u_e is known, corresponding values of x and y, may be found by using Blasius solutions for a cone or by assuming

any desired boundary layer profile. More features of this and subsequent subroutines may be obtained from the flow diagram, and the program. From the CINPUT, the MAIN program calls the CHAR subroutine which is used to evaluate conditions at a third point once the conditions at two other points not on the same characteristic are known (i.e., this is just a rotational axisymmetric characteristic program where the rotationality comes from the boundary layer profile).

Once all the points along a given expansion line are known, the shape of the shock originating from the tip of the cone may be found; this procedure is carried out in the CSHOCK subroutine.

When the expansion fan is completed the program goes on to the DIVST subprogram. This subroutine evaluates conditions on the dividing streamline by using previous first family characteristic lines that the computer has evaluated. This having been done, the computer goes back to the CHAR program to evaluate the next point on this new first family line.

After the expansion fan is completed, it will be seen that characteristic lines of the same family will tend to cross each other. Whenever this happens the location of the intersection is found and checked to see whether the assumed shock is strong or weak. If the shock is strong enough, formation of an imbedded shock is postulated, the necessary values upstream and downstream are evaluated and the program goes on to the next characteristic line in the flow. If it is weak shock (as determined by the $\Delta s/s$ criteria) evaluation of the characteristic line is continued with a new reference line used to evaluate the rest of the points on this line.

After every characteristic line is completed, at given values of x, M, p/p_{∞}, H, $p_{t_2}/p_{t_{\infty}}$, and y are evaluated so that profiles of these values vs. y may be plotted at different downstream locations.

MAIN PROGRAM

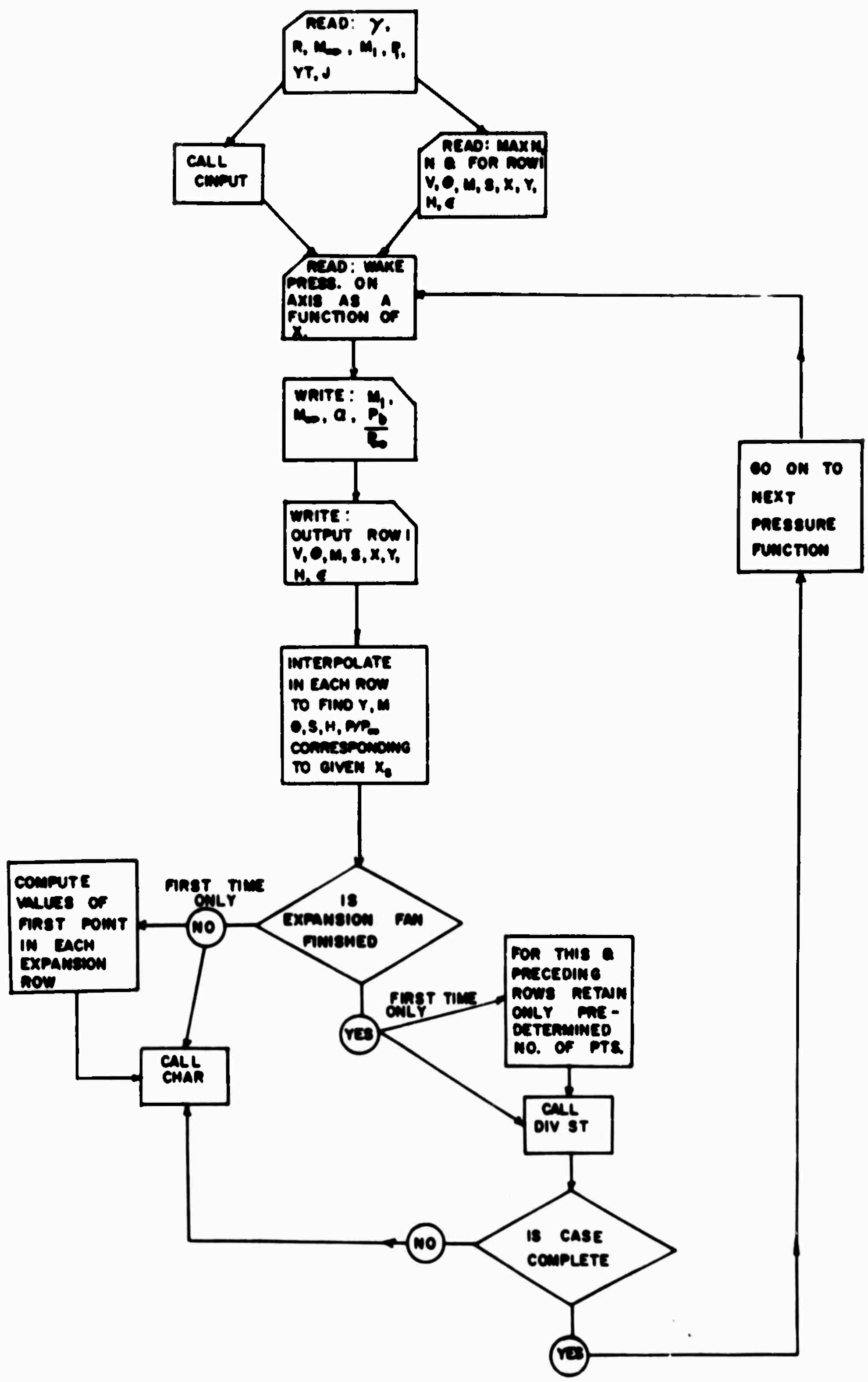


FIG. (A-1) INVISCID PROGRAM

SUBROUTINE CINPUT

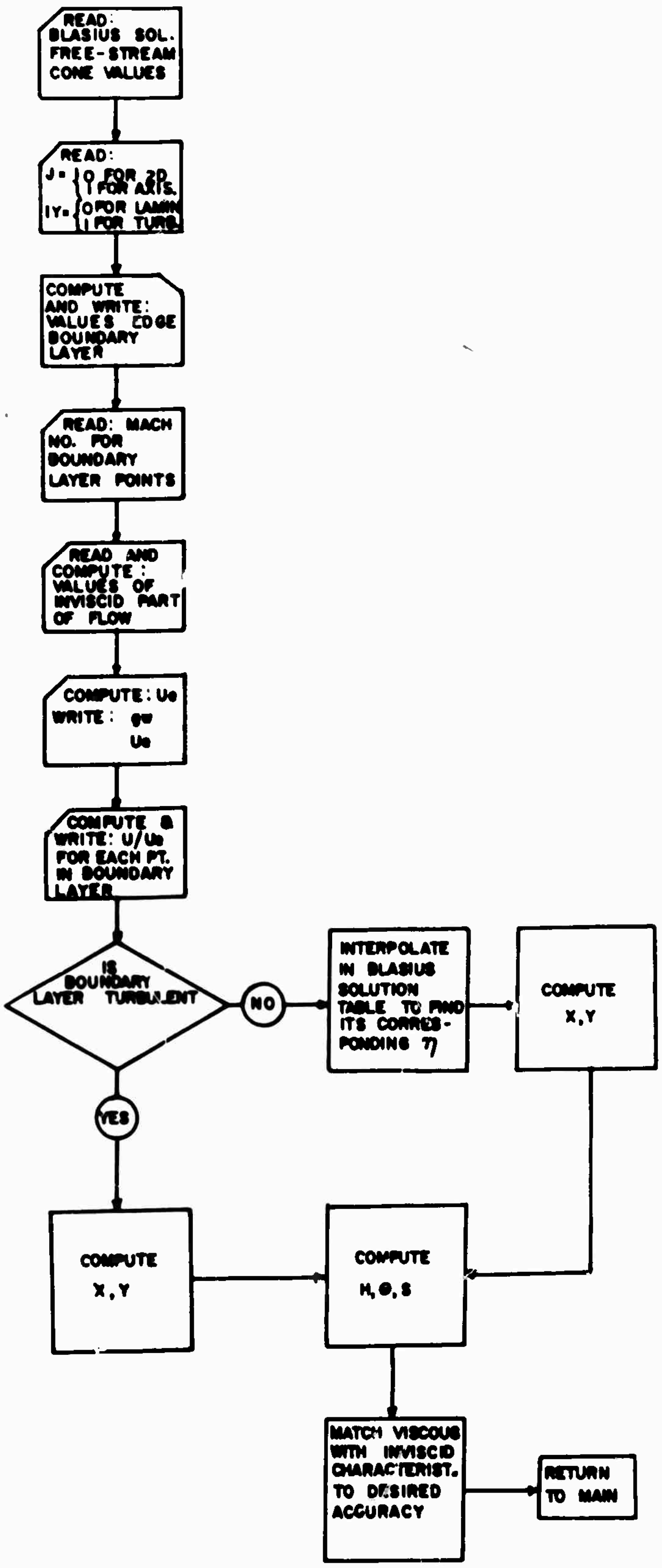


FIG. (A-2) INVISCID PROGRAM

SUBROUTINE CHAR

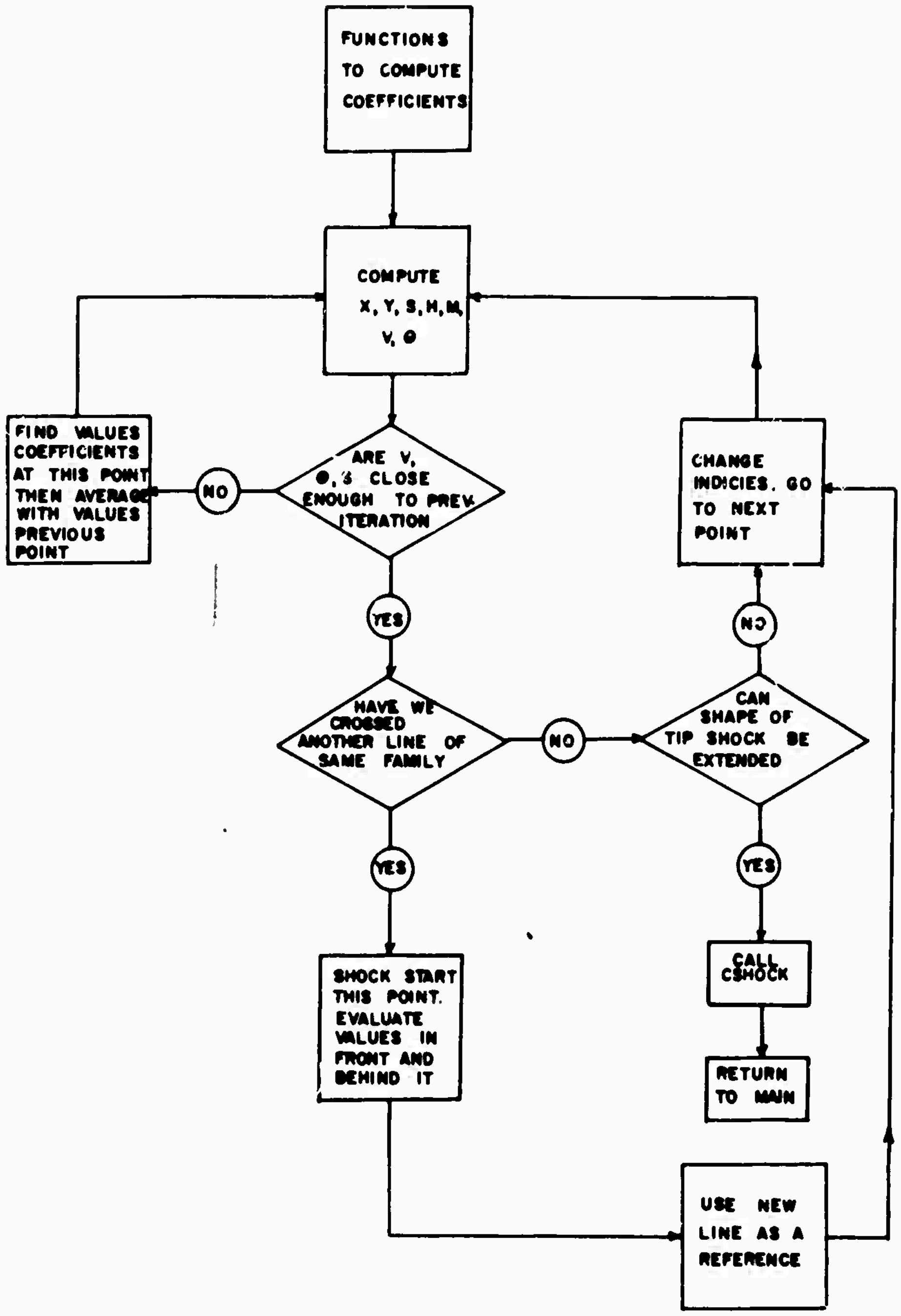


FIG. (A-3) INVISCID PROGRAM

SUBROUTINE CSHOCK

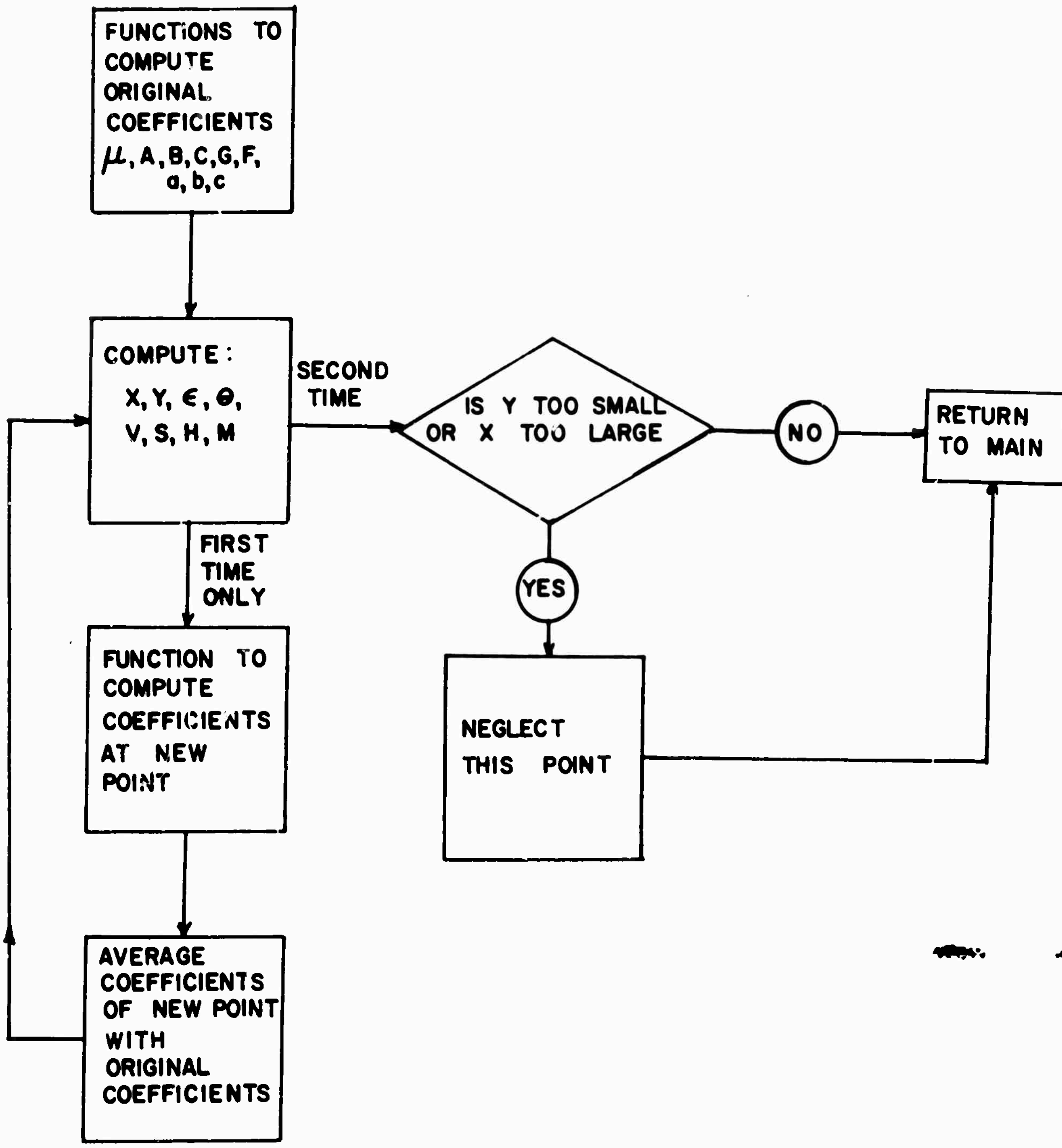


FIG. (A-4) INVISCID PROGRAM

SUBROUTINE DIV ST

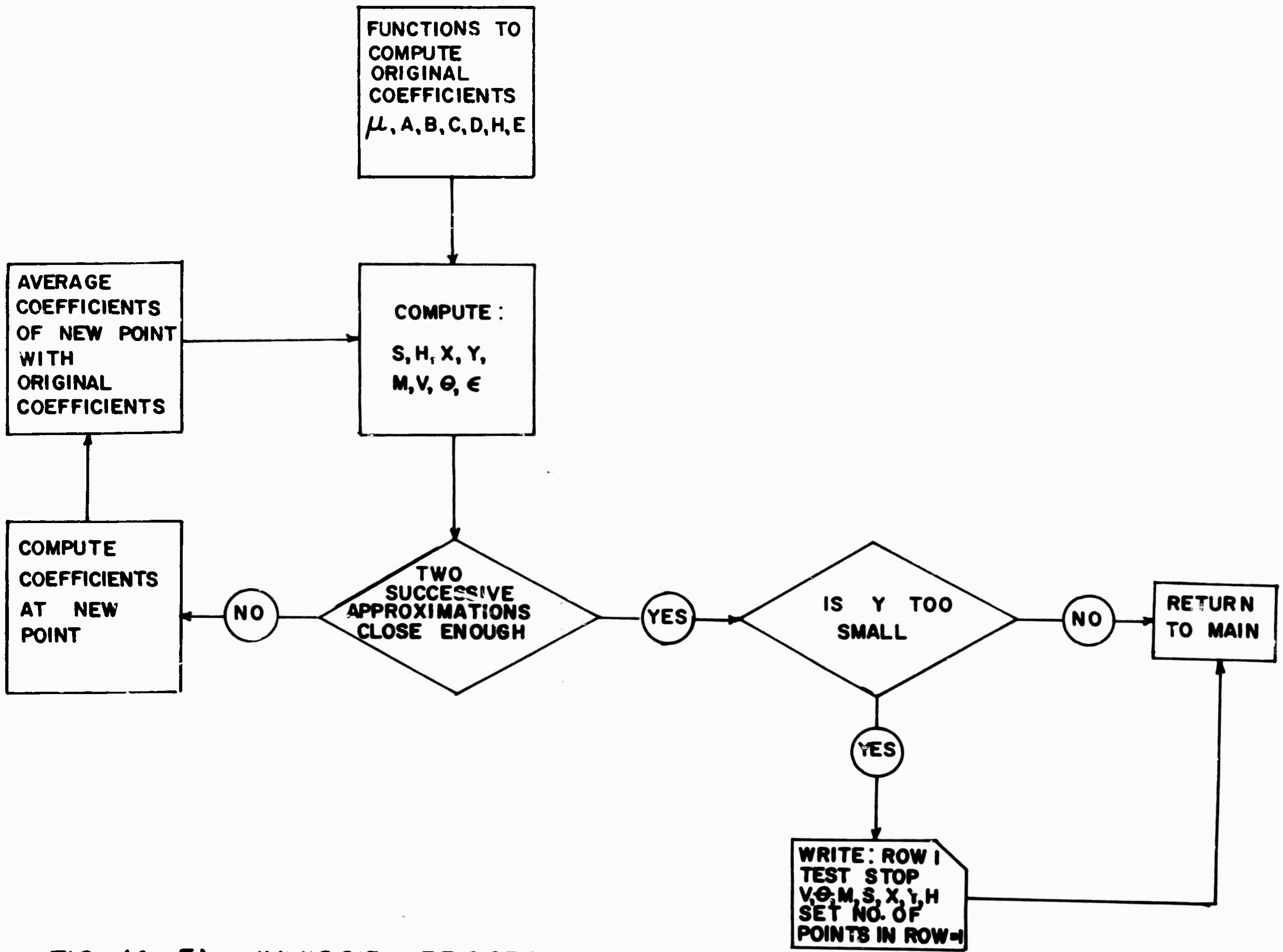


FIG. (A-5) INVISCID PROGRAM

COMPUTER PROGRAM FOR CHARACTERISTIC CALCULATION

```
      COMMON V(5,70),THETA(5,70),AM(5,70),S(5,70),X(5,70),Y(5,70),NL(5)
     1 ,EPI(5,70),AH(5,70),PM(5,70),LOC(5),XP(5,70),XX(5,150),
     1 GAM,RS,AMI,N,I,J,M,JAM,ALP,TBET,NP,KA,MAXN,MN,MC,IM,NPT,LPSH,
     1LL,TL,P1,EM1,YT,IDEL,IRED,KK,AL,A4,B4,C4,D4,E4,F4,IR,GW,SR,PSI,TSI
      DIMENSION UGUE(41),ETA(41),XZ(10)
      DIMENSION P2X(10),P2P(10)
      REWIND1
      REWIND2
     READ(5,370)GAM,RS,AMI,ALP,TBET
      READ(5,380) IN
      READ(5,380) INF
      READ(5,360) P1,EM1
      READ(5,340)IRED,IDEL
      READ(5,520) YT
      READ(5,520) XT
     READ(5,380)JAM
      READ(5,350) LL,TL
      READ(5,380) LN
      READ(5,490) (XZ(K),K=1,LN)
      READ(5,380)LPSH
     READ(5,380) KK
      IF(IN.EQ.1) GO TO 10
      READ(5,380)N
      READ(5,380)MAXN
     READ(5,390)(V(1,J),THETA(1,J),AM(1,J),S(1,J),X(1,J),Y(1,J),EPI(1,J
     1 ),AH(1,J),J=1,N)
      GO TO 20
10    CALL CINPUT
      MAXN=N+KK-1
20    INN=N
      IMAX=MAXN
      DO 30 J=2,N
30    PM(1,J)=(Y(1,J)-Y(1,J-1))/(X(1,J)-X(1,J-1))
      G2=(GAM-1.)/2.
      UN1=1.+G2*AMI**2
      PX=GAM/(GAM-1.)
40    KA=1
      IR=1
      N=INN
      MAXN=IMAX
      NRED=5
      MC=0
     NP=1
     M=0
      READ(5,480) A4,B4,C4,D4,E4,F4
      DL=AL/.0174532925
      TOC=TSI*GW
      SRT=12.*SR
      PSID=PSI/144.
      WRITE(6,400) AMI,TSI,PSID
      IF(JAM.EQ.1) WRITE(6,410) DL
      IF(JAM.EQ.0) WRITE(6,420) DL
      WRITE(6,430) SRT,TOC,A4,EM1
     I=1
     MN=0
      IM=KK-1
```

```
50    WRITE(6,450)
      WRITE(6,460) IR
     WRITE(6,470)(V(I,J),THETA(I,J),AM(I,J),S(I,J),X(I,J),Y(I,J),AH
     1(I,J),J=1,N)
      WRITE(6,440) EPI(I,N)
      WRITE(6,510)
      DO 80 LX=1,LN
      IF(XZ(LX).LT.X( I,1)) GO TO 80
      DO 60 LJ=1,50
      IF(X( I,LJ).EQ.0.) GO TO 80
      IF(XZ(LX).LT.X( I,LJ)) GO TO 70
60    CONTINUE
70    FL=(X( I,LJ)-XZ(LX))/(X( I,LJ)-X( I,LJ-1))
      FH=(XZ(LX)-X( I,LJ-1))/(X( I,LJ)-X( I,LJ-1))
      TV=THETA( I,LJ-1)*FL+THETA( I,LJ)*FH
      AMV=AM( I,LJ-1)*FL+AM( I,LJ)*FH
      TEMP = (1. + AMV**2/5.)
      SV=S( I,LJ-1)*FL+S( I,LJ)*FH
      VV=V( I,LJ-1)*FL+V( I,LJ)*FH
      YV=Y( I,LJ-1)*FL+Y( I,LJ)*FH
      HV=AH( I,LJ-1)*FL+AH( I,LJ)*FH
      PV=(UN1/(1.+G2*AMV**2))**PX*EXP(-SV)*HV**PX
      SIG = PV*TEMP*VV*YV*COS(TV)/SQRT(HV)
      PT2=((GAM+1.)/2.*AMV**2/UN1)**PX*((GAM+1.)/2./(GAM*AMV**2-G2))
     1 **(1./(GAM-1.))*PV
      WRITE(6,500) XZ(LX),IR,YV,SIG,AMV,VV,SV,PV,PT2,HV
      WRITE(2)XZ(LX),IR,YV,SIG,AMV,VV,SV,PV,PT2,HV
80    CONTINUE
      IF(X(I,1).GT.XT) CALL EXIT
      NU(I)=N
     I=I+1
      IR=IR+1
      IF(I.GT.3) GO TO 290
90    J=1
     N=N+1
     IF(N.GT.MAXN) GO TO 150
100   IF(IR.GT.KK) GO TO 160
      IF(INF.EQ.1) GO TO 280
      IF(I.GT.2) GO TO 110
      CM=GAM-1.
      GP=GAM+1.
      TF=THETA(1,1)
      AMF=AM(1,1)
      AMK      =SQRT(2./GM*((EM1**2*GM/2.+1.)*((P1/A4)**(GM/GAM))-1.))
      CNUL=SQRT(GP/GM)*ATAN(SQRT(GM/GP*(AMK**2-1.)))-ATAN(SQRT(AMK**2.
     1 -1.))
      CNUF=SQRT(GP/GM)*ATAN(SQRT(GM/GP*(AM(1 ,1)**2-1.)))-ATAN(SQRT
     1 (AM(1 ,1)**2.-1.))
      TKK          =CNUF+THETA(1,1)-CNUL
      XKK=KK-1
      SAT=SQRT(GP/GM)
110   X(I,1)=X(1,1)
      Y(I,1)=Y(1,1)
      S(I,1)=S(1,1)
      AH(I,1)=AH(1,1)
      EPI(I,1)=EPI(1,1)
```

```
      AM(I,1)=AMF
      XIK=IR-1
      THETA(I,1)=TF+(TKK-TF)*XIK/XKK
      CNU=CNUF+TF-THETA(I,1)
      DO 120 KI=1,100
      CNW=SAT*ATAN(SQRT((AM(I ,1)**2-1.)*GM/GP))-ATAN(SQRT(AM(I ,1)**2
     1 -1.))
      AM(I ,1)=AM(I ,1)+2.*(CNU-CNW)
      IF(ABS(CNU-CNW).LE..00001) GO TO 130
120   CONTINUE
130   V(I ,1)=SQRT(AM(I ,1)**2/(2./GM+AM(I ,1)**2))
140   CALL CHAR
      GO TO 50
150  N=MAXN
     MN=1
      GO TO 100
160   IF(NRED.EQ.0) GO TO 170
      CALL DIV ST
      IF(N.EQ.1) GO TO 40
      IF(N.EQ.2) GO TO 270
      GO TO 140
170   IF(IRED.LE.N) GO TO 180
      IRED=IRED-IDEL
      GO TO 170
180   ILP=IR-3
      REWIND1
      L=5
      LB=0
      DO 250 ILOOP=1,ILP
      READ(1)IW,NU(5),LOC(5)
      NUI=NU(5)
      READ(1)(V(5,J),THETA(5,J),AM(5,J),S(5,J),X(5,J),Y(5,J),EPI(5,J),
     1 AH(5,J),PM(5,J),XP(5,J),J=1,NUI)
      READ(1)(XX(5,IX),IX=1,IW)
      BACKSPACE1
      BACKSPACE1
      BACKSPACE1
190   K=1
      DO 200 J=1,IRED,IDEL
      V    (L,K)=V    (L,J)
      THETA(L,K)=THETA(L,J)
      AM   (L,K)=AM   (L,J)
      S    (L,K)=S    (L,J)
      X    (L,K)=X    (L,J)
      Y    (L,K)=Y    (L,J)
      AH   (L,K)=AH   (L,J)
      EPI  (L,K)=EPI  (L,J)
200   K=K+1
      INEX=IRED+1
      N=NU(L)
      IF(INEX.GT.N) GO TO 220
      DO 210 J=INEX,N
      V    (L,K)=V    (L,J)
      THETA(L,K)=THETA(L,J)
      AM   (L,K)=AM   (L,J)
      S    (L,K)=S    (L,J)
```

```
      X     (L,K)=X     (L,J)
      Y     (L,K)=Y     (L,J)
      AH    (L,K)=AH    (L,J)
      EPI   (L,K)=EPI   (L,J)
210   K=K+1
220   NU(L)=K-1
      N=NU(L)
      DO 230 J=2,N
230   PM(L,J)=(Y(L,J)-Y(L,J-1))/(X(L,J)-X(L,J-1))
      DO 240 IX=1,IW
240   XX(L,IX)=10.E+10
      IF(L.EQ.5) GO TO 260
250   CONTINUE
      LB=LB+1
      L=LB
      IF(L.LT.3) GO TO 190
      NRED=1
      MAXN=N
      I=3
      GO TO 160
260   WRITE(1)IW,NU(5),LOC(5)
      WRITE(1)(V(5,J),THETA(5,J),AM(5,J),S(5,J),X(5,J),Y(5,J),EPI(5,J),
     1 AH(5,J),PM(5,J),XP(5,J),J=1,N   )
      WRITE(1)(XX(5,IX),IX=1,IW)
      GO TO 250
270   IF(MC.NE.0) CALL SHOCK
      IF(NPT.EQ.1) CALL CHAR
      GO TO 50
280   READ(5,390) V(I,1),THETA(I,1),AM(I,1),S(I,1),X(I,1),Y(I,1),EPI(I,
     1 1),AH(I,1)
      GO TO 140
290   IW=IR-3
      NUI=NU(1)
      WRITE(1) IW,NUI,LOC(1)
      WRITE(1)(V(1,J),THETA(1,J),AM(1,J),S(1,J),X(1,J),Y(1,J),EPI(1,J),
     1 AH(1,J),PM(1,J),XP(1,J),J=1,NUI)
      WRITE(1)(XX(1,IX),IX=1,IW)
      DO 310 I=1,2
      N=NU(I+1)
      DO 300 J=1,N
      V     (I,J)= V     (I+1,J)
      THETA(I,J)= THETA(I+1,J)
      AM    (I,J)= AM    (I+1,J)
      S     (I,J)= S     (I+1,J)
      X     (I,J)= X     (I+1,J)
      Y     (I,J)= Y     (I+1,J)
      EPI   (I,J)= EPI   (I+1,J)
      AH    (I,J)= AH    (I+1,J)
      PM    (I,J)= PM    (I+1,J)
300   XP(I,J)=XP(I+1,J)
      LOC(I)=LOC(I+1)
      NU(I)=NU(I+1)
      IW=IW+1
      DO 310 IX=1,IW
310   XX(I,IX)=XX(I+1,IX    )
      DO 320 IX=1,IR
```

```
320   XX(3 ,IX)=0.
      I=3
      N=NU(3 )
      DO 330 J=1,N
      V(I,J)=0.
      THETA(I,J)=0.
      AM(I,J)=0.
      S(I,J)=0.
      X(I,J)=0.
      Y(I,J)=0.
      AH(I,J)=0.
      XP(I,J)=0.
      PM(I,J)=0.
330   EPI(I,J)=0.
      GO TO 90
340   FORMAT(2I5)
350   FORMAT(I3,E13.6)
360   FORMAT(2E18.8)
370  FORMAT(5E15.6)
380  FORMAT(I2)
390  FORMAT(4E18.8/4E18.8)
400   FORMAT(/////,1X,25HFREE STREAM MACH NUMBER =,F18.8,//,1X,36HFREE
     1STREAM STAGNATION TEMPERATURE =,F18.8,3X,15HDEGREES RANKINE,//1X,
     1 33HFREE STREAM STAGNATION PRESSURE =, F18.8,3X,3HPSI, /)
410   FORMAT(1X,20HHALF ANGLE OF CONE =,F18.8,3X,7HDEGREES, /)
420   FORMAT(1X,21HHALF ANGLE OF WEDGE =,F18.8,3X,7HDEGREES, /)
430   FORMAT(1X,24HRADIUS OF BASE OF CONE =,F18.8,3X,6HINCHES,//,1X,21H
     1TEMPERATURE OF CONE =,F18.8,3X,15HDEGREES RANKINE,//1X,30HPRESSURE
     1 BASE / PRESSURE INF =,F18.8,//,1X,49HINITIAL MACH NUMBER OF TRAIL
     1ING EDGE STREAMLINE =,F18.8)
440   FORMAT(100X,10HEPSILON = ,E18.8)
450  FORMAT(///,124H              V                  THETA                 M
     1                S                   X                   Y
     1H       )
460  FORMAT(///8H   ROW  ,I2///)
470  FORMAT(7E18.8)
480   FORMAT(6E13.6)
490   FORMAT(10F7.3)
500   FORMAT(2X,2HX=,F7.3,1X,I2,4X,8E13.4)
510   FORMAT(///,11X,3HROW,10X,1HY,11X,3HSIG,11X,1HM,12X,2HVV,11X,
     1 2HSV,10X,4HP/PI,9X,7HPT2/PTI,7X,2HHV)
520  FORMAT(E18.8)
     END
```

```
      SUBROUTINE CINPUT
      COMMON V(5,70),THETA(5,70),AM(5,70),S(5,70),X(5,70),Y(5,70),NL(5)
     1 ,EPI(5,70),AH(5,70),PM(5,70),LOC(5),XP(5,70),XX(5,150),
     1 GAM,RS,AMI,N,I,J,M,JAM,ALP,TBET,NP,KA,MAXN,MN,MC,IM,NPT,LFSH,
     1LL,TL,P1,EM1,YT,IDEL,IRED,KK,AL,A4,B4,C4,D4,E4,F4,IR,CW,SR,PSI,TSI
      DIMENSION UCUE(41),ETA(41)
      READ(5,190) (ETA(I),UCUE(I),I=1,41)
      READ(5,200)EME,GW,TSI,PSI,R,CPP,CI,AL,SR,TCT,RCR,AMIS
      READ(5,240)AP
      READ(5,220) IY
      GM=GAM-1.
      GP=GAM+1.
      XL=SR/SIN(AL)
      PI=1.+GM/2.*AMI**2
      PE=1.+GM/2.*EME**2
      TE=TSI*TCT/PI
      RE=RCR/PI**(1./GM)*PSI/(R*TSI)
      UE=AMIS*SQRT(GM/GP*2.*CPP*TSI)
      EU=2.27E-8*TE**1.5/(198.6+TE)
      REL=RE*UE*XL/EU
      XL=XL/SR
      AJA=JAM
      RR=2.*XL/SQRT((2.*AJA+1.)*REL)
      UE0=EME/AMI*SQRT(PI/PE)
      WRITE(6,230) REL,RE,EU,XL,UE,UE0
      READ(5,220) NC
      READ(5,240)(AM(1,J),J=1,NC)
      READ(5,220) NL
      J1=NC+1
      J2=NL+NC
     READ(5,210)(X(1,J),Y(1,J),THETA(1,J),S(1,J),AH(1,J),AM(1,J),J=J1,
     1 J2)
      DO 10 J=J1,J2
      EPI(1,J)=0.
      V(1,J)=SQRT(GM/GP)*AM(1,J)
10    AM(1,J)=SQRT((2./GM*V(1,J)**2)/(1.-V(1,J)**2))
      READ(5,240) EPI(1,J2)
      VE=SQRT(1./(1.+2./GM/EME**2))
      WRITE(6,290) GW,VE
      DO 80 J=1,NC
      EPI(1,J)=0.
      V(1,J)=SQRT(1./(1.+2./GM/AM(1,J)**2))
      VEV=(VE/V(1,J))**2
      UU=((1.-GW)+SQRT((1.-GW)**2+4.*GW*VEV))/(2.*VEV)
      WRITE(6,280) UU
      IF(IY.EQ.1) GO TO 180
     IF(UU.LT.UCUE(1)) GO TO 30
      IF(UU.GT.UCUE(41)) GO TO 30
      DO 20 I=1,41
      IF(UU.EQ.UCUE(I)) GO TO 40
      IF(UU.LT.UCUE(I)) GO TO 50
20    CONTINUE
30    WRITE(6,250) UU
      CALL EXIT
40    TA=ETA(I)
      GO TO 60
```

```
50    TA=ETA(I-1)+(ETA(I)-ETA(I-1))/(UCUE(I)-UCUE(I-1))*(UL-UCLE(I-1))
60    EX=1.-EXP(-TA)
      EXX=1.-EXP(-2.*TA)
      AY     =RR*(GW*PE*TA+PE*(1.-GW)*(TA-CI*EX)-(PE-1.)*(TA-2.*CI*EX+
     1 CI**2/2.*EXX))
      Y(1,J)=AY*CCS(AL)+1.0
70    AH(1,J)=GW+(1.-GW)*UU
      THETA(1,J)=AL
      S(1,J)=-2.*GAM/GM*(ALOG(AM(1,J)/EME)-ALOG(LU     ))+S(1,J1)
      UX=U(AM(1,J))
      AFF=AF
      AF=COTAN(THETA(1,J)+UX)
      IF(J.EQ.1) GC TO 170
      X(1,J)=X(1,J-1)+(AF+AFF)/2.*(Y(1,J)-Y(1,J-1))
80    CONTINUE
90    IF(X(1,ND).GT.X(1,J2)) GO TO 160
      DO 100 J=J1,J2
      IF(X(1,ND).LT.X(1,J)) GO TO 110
100   CONTINUE
110  ZK=Y(1,J-1)+(Y(1,J)-Y(1,J-1))/(X(1,J)-X(1,J-1))*(X(1,NC)-X(1,J-1))
      BE=Y(1,ND) - ZK
      WRITE(6,270) BE
      DO 120 J=J1,J2
      Y(1,J)=Y(1,J)*(1.+BE)
120   X(1,J)=X(1,J)*(1.+BE)
      IF(ABS(BE).GT..0000005) GC TC 90
      DO 130 J=J1,J2
      IF(X(1,J).GT.X(1,NC)) GO TO 140
130   CONTINUE
140   L=J1-1
      DO 150 K=J,J2
      L=L+1
      X(1,L)=X(1,K)
      Y(1,L)=Y(1,K)
      V(1,L)=V(1,K)
      S(1,L)=S(1,K)
      AH(1,L)=AH(1,K)
      AM(1,L)=AM(1,K)
      EPI(1,L)=EPI(1,K)
      THETA(1,L)=THETA(1,K)
150   CONTINUE
      N=ND+J2-J+1
      IF(IY.EQ.0) WRITE(6,300) NC
      IF(IY.EQ.1) WRITE(6,310) NC
      RETURN
160   WRITE(6,260) X(1,NC),X(1,J1),X(1,J2)
      CALL EXIT
170   X(1,1)=-AY*SIN(AL)
      IF(IY.EC.1) X(1,1)=-(UU**AP*.066*SR*EME**.824)/REL**.116
      GO TO 80
180   Y(1,J)=(LL**AP*.066*SR*EME**.824)/REL**.116*COTAN(AL)+1.C
      GO TO 70
190   FORMAT(F6.4,E13.6)
200   FORMAT (4E18.8)
210   FORMAT(3E18.8)
220   FORMAT(I2)
```

```
230   FORMAT(1H1,1X,4HREL=,E15.8,5HRHOE=,E15.8,4HMUE=,E15.8,2HL=,E15.8,
     1 3HUE=,E15.8,6HUE/UI=,E15.8)
240   FORMAT(E18.8)
250   FORMAT(1X,9HL OVER UE,E18.8,12HOUT OF RANGE)
260   FORMAT(1X,15HLAST DARK POINT,E18.8,24HNOT BETWEEN LIGHT POINTS,
     1 2E18.8)
270   FORMAT(2X,3HBE=,E18.8)
280     FORMAT (2X,3HUU=,E18.8)
290   FORMAT(/,1X,3HGW=,E18.8,3HVE=,E18.8,///)
300   FORMAT(/////,1X,  26HLAMINAR BOUNDARY LAYER ---,I5,16H POINTS ARE
     1 USED)
310   FORMAT(/////,1X,28HTURBULENT BOUNDARY LAYER ---,I5,16H POINTS ARE
     1 USED)
      END
```

```
      SUBROUTINE CHAR
      COMMON V(5,70),THETA(5,70),AM(5,70),S(5,70),X(5,70),Y(5,70),NL(5)
     1 ,EPI(5,70),AH(5,70),PM(5,70),LOC(5),XP(5,70),XX(5,150),
     1 GAM,RS,AMI,N,I,J,M,JAM,ALP,TBET,NP,KA,MAXN,MN,MC,IM,NPT,LPSH,
     1 LL,TL,P1,EM1,YT,IDEL,IRED,KK,AL,A4,B4,C4,D4,E4,F4,IR
      T(XX,YY)=COS(XX)/CCS(YY+XX)
      Z(XX,YY)=COS(XX)/COS(YY-XX)
       WRITE(6,520) (XX(I-1,I2),I2=1,IR)
      IF(NPT.EQ.1) GO TO 140
      GO=0.
     LIM=N-1+M
      IF(MC.GT.0) LIM=LIM-1
     INTEGERP
       DO 330 J=2,LIM
10    IF(GO.EQ.1..AND.XP(I,J-1).EQ.0.) GO TO 120
      IF(GO.NE.1.) GO TO 130
      IQ=II
      IF(II.LT.(IR-2)) GO TO 40
      IF((IR-II).EQ.2) IX=1
      IF((IR-II).EQ.1) IX=2
20    IQ=IQ-1
      IF(IQ.EQ.0) GO TO 400
      IF(XX(I,II).GT.XX(IX,IQ)) GO TO 20
      IX=IQ
      II=IX
      IF(II.LT.(IR-2)) GO TO 50
      IF((IR-II).EQ.2) IX=1
      IF((IR-II).EQ.1) IX=2
30    P=J+1
      IF(II.LT.KK) P=P+II-KK
      GO TO 140
40    IWHE=1
      GO TO 60
50    IWHE=2
60    LBACK=0
70    BACKSPACE1
      BACKSPACE1
      BACKSPACE1
      READ(1) IS,NU(5),LOC(5)
      IF(II.EQ.IS) GO TO 80
      BACKSPACE1
      LBACK=LBACK+1
      GO TO 70
80    NUI=NU(5)
      READ(1)(V(5,K),THETA(5,K),AM(5,K),S(5,K),X(5,K),Y(5,K),EPI(5,K),
     1 AH(5,K),PM(5,K),XP(5,K),K=1,NUI)
      READ(1)(XX(5,IX),IX=1,IS)
      IF(LBACK.EQ.0) GO TO 100
      DO 90LBA=1,LBACK
      READ(1)ITT,NU(4),LOC(4)
      NUI=NU(4)
      READ(1)(V(4,K),THETA(4,K),AM(4,K),S(4,K),X(4,K),Y(4,K),EPI(4,K),
     1 AH(4,K),PM(4,K),XP(4,K),K=1,NUI)
90    READ(1)(XX(4,IX),IX=1,ITT)
100   IX=5
      IF(IWHE.EQ.1) GO TO 20
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```
      GO TO 30
110   IX=I -1
      II=IR-1
      GO TO 170
120   P=P+1
      GO TO 140
130   IX=I -1
      II=IR-1
      P=J+NP-1
140   L=0
       WRITE(6,510) IR,J,II,P
150  L=L+1
160   IF(NPT.EQ.1) GO TO 110
170   IF(L.EQ.1) GO TO 450
180   HG=HX-GX
      X(I,J)=(Y(I,J-1)-Y(IX,P)+HX*X(IX,P)-GX*X(I,J-1))/HG
      Y(I,J)=(GX*HX*(X(IX,P)-X(I,J-1))+HX*Y(I,J-1)-GX*Y(IX,P))/HG
      DELX=X(I,J)-X(IX,P)
      DELZ=X(I,J)-X(I,J-1)
      XN=DELZ*ANX
      XM=DELX*SMAX
     SS=S(IX ,P)-S(I,J-1)
     DELH=AH(IX ,P)-AH(I,J-1)
     XNM=SS/(XN+XM)
      DX=DW*DELX
      S3=S(I,J)
      S(I,J)=S(I,J-1)+XNM*XN
      AH(I,J)=AH(I,J-1)+((DELH/(XN+XM))*XN)
      S1=V(I,J)
      IF(AH(I,J).LT.0.) AH(I,J)=(AH(I-1,J)+AH(I-1,J+1))/2.
190   V(I,J)=(AX*V(I,J-1)+AW*V(IX,P)+THETA(IX,P)-THETA(I,J-1)+BX*DELZ
    1 +DX+SS*(CX*DELX*SMAX-DELZ*ANX*CW)/(XN+XM)+DELH/(2.*(XN+XM))*(TX*
    1 DELZ/(AH(I,J-1)*V(I,J-1)**2)-ZW*DELX/(AH(IX,P)*V(IX,P)**2)))/(AX
    1 *SQRT(AH(I,J)/AH(I,J-1))+AW*SQRT(AH(I,J)/AH(IX,P)))
      S2=THETA(I,J)
      THETA(I,J)=THETA(IX,P)+AW*(V(IX,P)-SQRT(AH(I,J)/AH(IX,P))*V(I,J))
    1 +DX+CX*XM*XNM-ZW*DELX*DELH/(2.*V(IX,P)**2*AH(IX,P)*(XN+XM))
      IF(S2.NE.0..AND.L.EQ.1) GO TO 250
      IF(V(I,J).LT.0..OR.V(I,J).GT.1.) GO TO 390
     AM(I,J)=SQRT(2./(GAM-1.))*SQRT(V(I,J)**2/(1.-V(I,J)**2))
      IF(ABS((V (I,J)-S1)/V (I,J)).LT.TL) GO TO 240
200  EPI(I,J)=0.
      PM(I,J)=(Y(I,J)-Y(I,J-1))/(X(I,J)-X(I,J-1))
      IF(IR.LE.KK) GO TO 220
210   IF(L.GT.LL) GO TO 420
220   IF(AM(I,J).LT.1.) AM(I,J)=(AM(I-1,J)+AM(I-1,J+1))/2.
      IF(L.EQ.1) GO TO 460
230   UX=U(AM(I,J))
      AX=.5*(AZ+A(UX,V(I,J)))
      AW=.5*(AY+A(UX,V(I,J)))
      DW=.5*(DY+D(JAM,UX,THETA(I,J),Y(I,J)))
      BX=.5*(BZ+B(JAM,UX,THETA(I,J),Y(I,J)))
      CW=.5*(CY+C(UX,GAM,RS))
      CX=.5*(CZ+C(UX,GAM,RS))
      ZW=.5*(ZZ+Z(UX,THETA(I,J)))
      TX=.5*(TZ+T(UX,THETA(I,J)))
```

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      GX=.5*(GZ+TAN(THETA(I,J)+UX))
      HX=.5*(HZ+TAN(THETA(I,J)-UX))
      IF(IR.LE.KK.AND.L.GT.LL) GO TO 310
     GO TO 150
240   IF(V(I,J).GE.1.) GO TO 210
      IF(ABS((THETA(I,J)-S2)/THETA(I,J)).GE.TL.OR.ABS((S(I,J)-S3)/S(I,J
     1 )).GE.TL) GO TO 210
      IF(Y(I,J).LT.0..OR.Y(I,J).LT.TBET) GO TO 400
250   PM(I,J)=(Y(I,J)-Y(I,J-1))/(X(I,J)-X(I,J-1))
      IF(L.EQ.1) GO TO 260
      IF(IR.LE.KK) GO TO 310
      IF(PM(I,J).GT.PM(IX ,P  )) GO TO 260
      GO TO 310
260   YPM      = (PM(I,J)*Y(IX ,P-1) - PM(IX ,P)*Y(I,J-1) + PM(I,J)*
     1 PM(IX ,P)*(X(I,J-1) - X(IX ,P-1)))/(PM(I,J) -PM(IX ,P))
      XP(I,J)=X(I,J-1)+((X(I,J)-X(I,J-1))/(Y(I,J)-Y(I,J-1)))*(YPM-Y(I,J
     1 -1))
      IF(L.EQ.1) GO TO 270
      IF(XP(I,J).LT.X(IX ,P)) GO TO 270
      XP(I,J)=0.
      GO TO 310
270   WRITE(6,480)IR,J,YPM
      LOC(I)=6+L2
      IF(L2.LE.0) L2=1
      MC=2
      IF(XP(I,J).LT.X(I,J-1)) GO TO 370
      WRITE(6,490) IR,J,XP(I,J)
      XX(I,II)=XP(I,J)
      YA=Y(I,J-1)-Y(IX,P-1)
      YB=Y(IX,P)-Y(IX,P-1)
      YC=YPM-Y(IX,P-1)
      XA=X(IX,P)-X(IX,P-1)
      XB=X(I,J-1)-X(IX,P-1)
      XC=XP(I,J)-X(IX,P-1)
      DEN=XA*YA-YB*XB
      DFXT=((THETA(IX,P)-THETA(IX,P-1))*YA-YB*(THETA(I,J-1)-THETA(IX,
     1  P-1)))/DEN
      DFXH=((AH(IX,P)-AH(IX,P-1))*YA-YB*(AH(I,J-1)-AH(IX,P-1)))/DEN
      DFXV=((V (IX,P)-V (IX,P-1))*YA-YB*(V (I,J-1)-V (IX,P-1)))/DEN
      DFXS=((S (IX,P)-S (IX,P-1))*YA-YB*(S (I,J-1)-S (IX,P-1)))/DEN
      DFYT=(XA*(THETA(I,J-1)-THETA(IX,P-1))-XB*(THETA(IX,P)-THETA
     1 (IX,P-1)))/DEN
      DFYH=(XA*(AH(I,J-1)-AH(IX,P-1))-XB*(AH(IX,P)-AH(IX,P-1)))/DEN
      DFYV=(XA*(V (I,J-1)-V (IX,P-1))-XB*(V (IX,P)-V (IX,P-1)))/DEN
      DFYS=(XA*(S (I,J-1)-S (IX,P-1))-XB*(S (IX,P)-S (IX,P-1)))/DEN
      THETA(IX,P)=THETA(IX,P-1)+DFXT*XC+DFYT*YC
      V(IX,P)=V(IX,P-1)+DFXV*XC+DFYV*YC
      S(IX,P)=S(IX,P-1)+DFXS*XC+DFYS*YC
      AH(IX,P)=AH(IX,P-1)+DFXH*XC+DFYH*YC
      AM(IX ,P)=SQRT(2./(GAM-1.))*SQRT(V(IX ,P)**2/(1.-V(IX ,P)**2))
      YA=Y(I,J)-Y(I,J-1)
      YB=Y(IX,P-1)-Y(I,J-1)
      YC=YPM-Y(I,J-1)
      XA=X(IX,P-1)-X(I,J-1)
      XB=X(I,J)-X(I,J-1)
      XC=XP(I,J)-X(I,J-1)
```

```
      DEN=XA*YA-YB*XB
      DFXT=((THETA(IX,P-1)-THETA(I,J-1))*YA-(THETA(I,J)-THETA(I,J-1))*
     1 YB)/DEN
      DFXV=((V (IX,P-1)-V (I,J-1))*YA-(V (I,J)-V (I,J-1))*YB)/DEN
      DFXS=((S (IX,P-1)-S (I,J-1))*YA-(S (I,J)-S (I,J-1))*YB)/DEN
      DFXH=((AH(IX,P-1)-AH(I,J-1))*YA-(AH(I,J)-AH(I,J-1))*YB)/DEN
     DFYT=(XA*(THETA(I,J)-THETA(I,J-1))-(THETA(IX,P-1)-THETA(I,J-1))*XB
     1 )/DEN
     DFYV=(XA*(V (I,J)-V (I,J-1))-(V (IX,P-1)-V (I,J-1))*XB)/DEN
     DFYS=(XA*(S (I,J)-S (I,J-1))-(S (IX,P-1)-S (I,J-1))*XB)/DEN
     DFYH=(XA*(AH(I,J)-AH(I,J-1))-(AH(IX,P-1)-AH(I,J-1))*XB)/DEN
      THETA(I,J)=THETA(I,J-1)+DFXT*XC+DFYT*YC
      V    (I,J)=V    (I,J-1)+DFXV*XC+DFYV*YC
      S    (I,J)=S    (I,J-1)+DFXS*XC+DFYS*YC
      AH   (I,J)=AH   (I,J-1)+DFXH*XC+DFYH*YC
      IF(V(I,J).GT.1.) GO TO 440
      AM(I,J)=SQRT(2./(GAM-1.))*SQRT(V(I,J)**2/(1.-V(I,J)**2))
 280  Y(I,J)=YPM
      X(I,J)=XP(I,J)
      PM(I,J)=(Y(I,J)-Y(I,J-1))/(X(I,J)-X(I,J-1))
      GP=GAM+1.
      GM=(GAM-1.)/2.
      AM2=AM(IX,P)**2
      DL=0.
      IF(AM(I,J)**2-1..LT.0.) AM(I,J)=1.2
 290  ANS=                    GM/(GAM*AM2)+GP/(2.*GAM*AM2)*((1.+GM*AM2)/
     1 (1.+GM*AM(I,J)**2))**(GAM/(GAM-1.))*(1.-(GAM*AM(I,J)**2)/
     1 SQRT(AM(I,J)**2-1.)*DL+DL**2*GAM*AM(I,J)**2*(GP*AM(I,J)**4-4.*(AM
     1 (I,J)**2-1.))/((AM(I,J)**2-1.)**2*4.))*EXP(S(IX,P)-S(I,J))
      IF(ANS.LT.0.) GO TO 430
      EPI(I,J)=ARSIN(SQRT(ANS))+THETA(IX,P)
 300  ABC=ABS((S(I,J)/S(IX,P)-1.)*100.)
 305  IF(ABC.LT.YT) GO TO 410
      GO TO 370
 310 GO TO (320,330), KA
 320 IF(X(I,J).GT.ALP) GO TO 360
 33   CONTINUE
      IF((NU(IX)-1).GT.N) GO TO 470
      DO 340IJ=1,II
 340  XX(I,IJ)=10.E+10
      NPT=0
      IF(MC.GT.0) CALL SHOCK
      IF(NPT.EQ.1) GO TO 10
     IF(M.GT.0) GO TO 350
     IF(MN.GT.0) GO TO 350
      CALL CSHOCK
 350 RETURN
 360 KA=2
 370 N=J
 380  M=1
     MAXN=N
      LI=II-1
      DO 385IJ=1,LI
 385  XX(I,IJ)=10.E+10
     GO TO  350
 390  AM(I,J)=(AM(I-1,J)+AM(I-1,J+1))/2.
```

```
      GO TO 200
400   N=J-1
      GO TO 380
410   MC=0
      WRITE(6,500) ABC
      GO=1.
      GO TO 310
420   IF(IR.LE.KK) GO TO 400
      WRITE(6,530) IR,J
      GO TO 400
430   EPI(I,J)=THETA(I-1,J+1)
      GO TO 300
440   AM(I,J)=(AM(IX,P-1)+AM(IX,P))/2.
      GO TO 280
450   UX=U(AM(I,J-1))
      UW=U(AM(IX,P))
      AX=A(UX,V(I,J-1))
      AW=A(UW,V(IX,P))
      BX=B(JAM,UX,THETA(I,J-1),Y(I,J-1))
      ANX=AN(UX,THETA(I,J-1))
      CW=C(UX,GAM,RS)
      SMAX=SMA(UW,THETA(IX,P))
      CX=C(UW,GAM,RS)
      DW=D(JAM,UW,THETA(IX,P),Y(IX,P))
      ZW=Z(UW,THETA(IX,P))
      TX=T(UX,THETA(I,J-1))
      GX=TAN(THETA(I,J-1)+UX)
      HX=TAN(THETA(IX,P)-UW)
      GO TO 180
460   AZ=AX
      AY=AW
      BZ=BX
      CY=CW
      CZ=CX
      DY=DW
      ZZ=ZW
      TZ=TX
      GZ=GX
      HZ=HX
      GO TO 230
470   LIM=LIM+1
      J=LIM
      N=N+1
      MAXN=N
      GO TO 10
480   FORMAT(10X,4HYPM(,I3,1H,I3,1H),E20.8)
490   FORMAT(10X,4HXPM(,I3,1H,I3,1H),E20.8)
500   FORMAT(20X,E18.8)
510   FORMAT(5X,4I10)
520   FORMAT (1X,13E10.4)
530   FORMAT(1X,29HWE HAVE NOT CONVERGED FOR ROW,I5,5HPOINT,I5)
      END
```

```
      SUBROUTINE CSHOCK
      COMMON V(5,70),THETA(5,70),AM(5,70),S(5,70),X(5,70),Y(5,70),NL(5)
     1 ,EPI(5,70),AH(5,70),PM(5,70),LOC(5),XP(5,70),XX(5,150),
     1 GAM,RS,AMI,N,I,J,M,JAM,ALP,TBET,NP,KA,MAXN,MN,MC,IM,NPT,LFSH,
     1 LL,TL,P1,EM1,YT,IDEL,IREC,KK,AL,A4,B4,C4,D4,E4,F4,IR
      J=N
299   UX=U(AM(I,J-1))
      AX=A(UX,V(I,J-1))
      BX=B(JAM,UX,THETA(I,J-1),Y(I,J-1))
      CX=C(UX,GAM,RS)
      GX=TAN(THETA(I,J-1)+UX)
      FX=TAN(EPI(I-1,J-1))
      GM1=GAM-1.
      AMIS=AMI**2
      GM=GM1/2.*AMIS
      GP1=GAM+1.
      BET=EPI(I-1,J-1)-THETA(I-1,J-1)
      SA=-V(I-1,J-1)*SIN(BET)*COS(BET)*(FX/TAN(BET)+TAN(BET)/FX-
     1 (2.*GM1)/GP1)
      COM=(AMI*SIN(EPI(I-1,J-1)))**2
      COMS=(COM-1.)**2
      SB=-SIN(2.*THETA(I-1,J-1))/SIN(2.*EPI(I-1,J-1))+((GP1*AMI**2
     1 *COM*SIN(THETA(I-1,J-1))**2)/COMS)
      SC=RS/FX*COMS/((COM-(GM1/(2.*GAM)))*(COM*(GM1/2.)+1.))
      L=0
310   L=L+1
      FG=FX-GX
      X(I,J)=(Y(I,J-1)-Y(I-1,J-1)+FX*X(I-1,J-1)-GX *X(I,J-1))/FG
      Y(I,J)=(FX*GX*(X(I-1,J-1)-X(I,J-1))+FX*Y(I,J-1)-GX*Y(I-1,J-1))/FG
      DELX=X(I,J)-X(I,J-1)
      EPI(I,J)=EPI(I-1,J-1)+(AX*(V(I,J-1)-V(I-1,J-1))-THETA(I,J-1)+
     1 THETA(I-1,J-1)+CX*(S(I,J-1)-S(I-1,J-1))+BX*DELX)/(SA*AX-SB+SC*CX)
      SEP=SIN(EPI(I,J))
      EM=AMIS*SEP**2
      THETA(I,J)=ATAN((AMIS*SIN(2.*EPI(I,J))-2.*COS(EPI(I,J))/SEP)/
     1(2.+AMIS*(GAM+COS(2.*EPI(I,J)))))
      VT=1.-((4.*(EM-1.)*(GAM*EM+1.)))/(GP1**2*AMIS*EM)
      IF(VT.LT.0.) GO TO 300
      V(I,J)=SQRT(GM/(1.+GM))*SQRT(VT)
      IF((2.*GAM*EM).LT.GM1) GO TO 300
      S(I,J)=RS/GM1*ALOG((2.*GAM*EM-GM1)/GP1)-GAM*RS/GM1*
     1ALOG(GP1*EM/(GM1*EM+2.))
      AH(I,J)=AH(I-1,J-1)
      IF(V(I,J).GT.1.) GO TO 300
      AM(I,J)=SQRT(2./GM1)*SQRT(V(I,J)**2/(1.-V(I,J)**2))
      IF(L.EQ.1) GO TO 340
      IF(S(I,J).LE.0.0.OR.X(I,J).LE.0.0) GO TO 300
      IF(AM(I,J).LT.1.) GO TO 300
320   IF(Y(I,J).LT.0..OR.Y(I,J).LT.TBET) GO TO 300
      PM(I,J)=(Y(I,J)-Y(I,J-1))/(X(I,J)-X(I,J-1))
      IF(X(I,J).GT.ALP) GO TO 330
      RETURN
330   M=1
      MAXN=N
      RETURN
```

```
300   N=J-1
      GO TO 330
340   AZ=AX
      BZ=BX
      CZ=CX
      GZ=GX
      FZ=FX
350   UX=U(AM(I,J))
      AX=.5*(AZ+A(UX,V(I,J)))
      BX=.5*(BZ+B(JAM,UX,THETA(I,J),Y(I,J)))
      CX=.5*(CZ+C(UX,GAM,RS))
      FX=.5*(FZ+TAN(EPI(I,J)))
      GX=.5*(GZ+TAN(THETA(I J)+UX))
     GO TO 310
     END
```

```
      SUBROUTINE CIV ST
      COMMON V(5,70),THETA(5,70),AM(5,70),S(5,70),X(5,70),Y(5,70),NL(5)
     1 ,EPI(5,70),AH(5,70),PM(5,70),LOC(5),XP(5,70),XX(5,150),
     1 GAM,RS,AMI,N,I,J,M,JAM,ALP,TBET,NP,KA,MAXN,MN,MC,IM,NPT,LFSH,
     1 LL,TL,P1,EM1,YT,IDEL,IRED,KK,AL,A4,B4,C4,D4,E4,F4,IR
      J=1
      UX=U(AM(I-1,2))
      AX=A(UX,V(I-1,2))
      CX=C(UX,GAM,RS)
      DX=D(JAM,UX,THETA(I-1,2),Y(I-1,2))
      HX=TAN(THETA(I-1,2)-UX)
      EX=TAN(THETA(I-1,1))
      N=N-NP+MC
      M=1
      NP=2
      GM=GAM-1.
      L=0
  110 L=L+1
      S(I,1)=S(I-1,1)
      AH(I,1)=AH(I-1,1)
      DEN=EX-HX
      X1=X(I,1)
      X(I,1)=(Y(I-1,2)-Y(I-1,1)+EX*X(I-1,1)-HX*X(I-1,2))/DEN
      DELX=X(I,1)-X(I-1,2)
      Y(I,1)=(EX*HX*(X(I-1,1)-X(I-1,2))+EX*Y(I-1,2)-HX*Y(I-1,1))/DEN
      XQ=X(I,1)
      AM(I,1)=SQRT(2./GM*((EM1**2*GM/2.+1.)*((P1/P2(XC,A4,B4,C4,D4,E4,F4
     1 ))**(GM/GAM))-1.))
      V(I,1)=SQRT(AM(I,1)**2/(2./GM+AM(I,1)**2))
      T1=THETA(I,1)
      THETA(I,1)=THETA(I-1,2)-AX*(V(I,1)*SQRT(AH(I,1)/AH(I-1,2))-V(I-1,
     1  2))+DX*DELX-CX*(S(I-1,1)-S(I-1,2))+AX*(AH(I,1)-AH(I-1,2))/
     1 (V(I-1,2)*(AH(I,1)+AH(I-1,2)))
      EPI(I,1)=0
      IF(ABS((X1-X(I,1))/X(I,1)).LT..0001.AND.ABS((T1-THETA(I,1))/
     1  THETA(I,1)).LT..0001) GO TO 120
      IF(L.GT.50) GO TO 115
      IF(L.NE.1) GO TO 180
      AZ=AX
      CZ=CX
      DZ=DX
      HZ=HX
      EZ=EX
  180 UX=U(AM(I,1))
      AX=.5*(AZ+A(UX,V(I,1)))
      CX=.5*(CZ+C(UX,GAM,RS))
      DX=.5*(DZ+D(JAM,UX,THETA(I,1),Y(I,1)))
      HX=.5*(HZ+TAN(THETA(I,1)-UX))
      EX=.5*(EZ+TAN(THETA(I,1)))
      GO TO 110
  115 WRITE(6,160)
  120 IF(Y(I,1).LT.0..OR.Y(I,1).LT.TBET) GO TO 130
      IF(N.EQ.1) GO TO 130
      RETURN
  130 WRITE(6,140)I
      WRITE(6,150)V(I,J),THETA(I,J),AM(I,J),S(I,J),X(I,J),Y(I,J),
```

```
     1AH (I,J)
      N=1
      RETURN
140  FORMAT(///8H   ROW  ,I2,15H     TEST  STOP///)
150  FORMAT(7E18.8)
160   FORMAT(40X,40HWE HAVE NOT CONVERGED ON THE FIRST POINT)
     END
```

APPENDIX B

PROGRAM FOR VISCOUS REGION

Once the subsonic portion of the boundary layer is subdivided into n strips, the flow in each strip is assumed to be governed by the one-dimensional equations with friction and heat transfer. The resulting system of equations consist of "3n" nonlinear, ordinary differential equations. These equations are connected through the boundary conditions on heat transfer, q, and friction, c_f (Fig. 6). Once the starting conditions, pressure distribution, and shape of basic streamline are known, the equations are solved simultaneously by the Runge-Kutta method.

At every station station x it is therefore possible to find an area consistent with the flow conditions in each stream tube in the flow field. Since the basic streamline and the area for the viscous layer are now known, the location of the Dividing Streamline can be obtained. The point at which the Dividing Streamline intersects the axis is the location of the rear stagnation point.

Downstream of the rear stagnation region the inviscid characteristic and viscous layer analyses must be such that the basic streamline is located at radial location where the outer streamline of the viscous layer exactly coincides with the inner streamline of the inviscid flow field. If at any x station this condition is not satisfied, then the assumed pressure along the basic streamline is changed until the above condition is satisfied.

For details of the viscous layer program, one may consult the flow chart (Fig. B-1) and the program.

SUBROUTINE VISCOUS

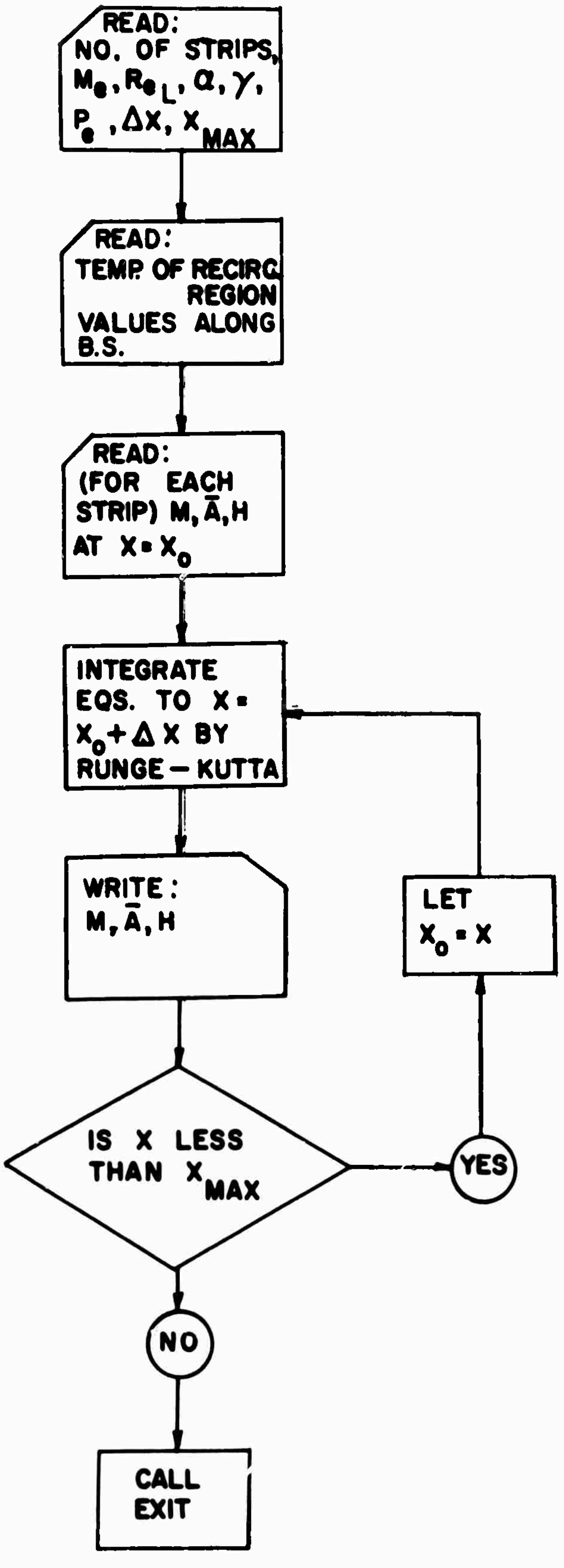

FIG. (B-1) VISCOUS PROGRAM

COMPUTER PROGRAM FOR SHEAR LAYER

```
      SUBROUTINE RK(X)
      DIMENSION S(20),F(20),H(20),P(20),U(20),A(50),B(100),R(22),SH(21)
     1 ,DN(20),Q(21),RX(20),XR(20),TX(20),AH(80),AS(80),AF(80),C(20),
     2 CF(20),FF(20),HF(20),V(20),CCF(20)
      DHX1(H,F,C)=-DX*((AME*SQRT(H)*6.2831853)*PE/(SQRT(F)*PF*C)*(GHF*
     1    SF-QLF*RF))
      DCX(H,F,C)=DX*(C*((1.-F)*UF/(GAM*PF*F)+(HX/H)*(1.+GMD*F)
     1 +((1.+GM*F)*CFT    /ABS(SF-RF))))
      DFX(H,F,C)=DX*(-F*(1.+GMD*F)*(2.*UF/(GAM*PF*F)+HX/H+CFT*2./ABS(SF
     1 -RF)))
      READ(5,150) NQ
      READ(5,170)     AME,REL,ALP,GAM,PE,TH
      READ(5,155) DX,AXX,E1,E2
      READ(5,155) VL,SHO,VO
      Z=0.
      NQD=NQ/3
      READ(5,170) (C(I),I=1,NQD)
      READ(5,170) (F(I),I=1,NQD)
      READ(5,170) (H(I),I=1,NQD)
      READ(5,150) IY
      IF(IY.NE.0) GO TO 147
      WRITE(6,270)
5     READ(5,150) SAME
      LIM=5*NQD
      IF(SAME.NE.0.) GO TO 7
      LIM=5
      NX=1
7     READ(5,170) (A(I),I=1,LIM)
      LI =2*LIM
      READ(5,170) (B(I),I=1,LI )
      READ(5,150) IXR
      READ(5,170) (XR(I),RX(I),TX(I),I=1,IXR)
      WRITE(6,250) E1,E2
      WRITE(6,255)A(1),B(1),B(2),B(2)
      WRITE(6,260) (A(I-1),A(I),B(2*I-1),B(2*I),B(2*I),I=2,LIM)
      WRITE(6,265) (XR(I),RX(I),TX(I),I=1,IXR)
      NQDP=NQD+1
      NQDM=NQD-1
      NW=NQD*4
      NX=NQD*5
      PH=3.14159265
      PCF=2.*PE*COS(ALP)/SIN(ALP)/REL
      S(NQD)=1.
      CCF(NQD)=0.
      GM=GAM-1.
      GMD=GM/2.
      HN=1.+GMD*AME**2
      EL=2./(REL*SIN(ALP)*(HN))
      XM=-1.
      IXF=0
      AB=COS( TH)
11    DO 111 I=1,IXR
      IF(X.LE.XR(I)) GO TO 112
111   CONTINUE
      I=IXR
      GO TO 113
```

```
112   IF(X.EQ.XR(I).OR.X.LT.XR(1)) GO TO 113
      R(1)=RX(I-1)+(RX(I)-RX(I-1))/(XR(I)-XR(I-1))*(X-XR(I-1))
       TTH=TX(I-1)+(TX(I)-TX(I-1))/(XR(I)-XR(I-1))*(X-XR(I-1))
      R(1)=R(1)/COS(TTH)
      GO TO 114
113   R(1)=RX(I)/COS(TX(I))
      TTH=TX(I)
114   IF(XM.GT.0.) R(1)=0.
      IF(IY.NE.0) R(1)=0.
      XT=X
      IW=1
      DO 12 I=1,NQD
      CF(I)=C(I)
      FF(I)=F(I)
12    HF(I)=H(I)
      ZF=Z
      IF(XM.LT.0.) GO TO 141
      IF(X.EQ.0.) Z=0.
25    SH(NQDP)=E1+E2*Z
      DO 70 I=1,NQD
      IF(HF(I).LT.0.) RETURN
      IF(FF(I).LT.0.) RETURN
      SH(I)=HN*HF(I   )/(1.+GMD*FF(I   ))
      V(I)=SQRT(FF(I)*SH(I))/AME
      DN(I)= S(I)-R(I)
      IF(IY.NE.0) SH(NQDP)=SHO
70    Q(I)=EL*SQRT(SH(I-1))*(SH(I)-SH(I-1))/(DN(I-1)+DN(I))
      Q(NQDP)=EL*SQRT(SH(NQD))*(SH(NQDP)-SH(NQD))/(DN(NQD)*2.)
      IF(XM.GT.0.) Q(NQDP)=0.
      IF(IY.NE.0) Q(NQDP)=0.0
      IF(IY.NE.0) Q(I)=-Q(I)
      Q(1)=EL*SQRT(SHO)*(SH(1)-SHO)/(DN(1)*2.0)
      IF(IY.NE.0) Q(1)=0.0
      IF(IXF.LT.1) GO TO 1391
      IF(IW.EQ.3) GO TO 120
      K=1
      Z=ZF+DX*COS(TTH)
      L=0
      DO 100 I=1,NX,5
      L=L+1
      IF(Z.LE.A(I)) J=K
      IF(A(I   ).LT.Z.AND.Z.LE.A(I+1)) J=K+2
      IF(A(I+1).LT.Z.AND.Z.LE.A(I+2)) J=K+4
      IF(A(I+2).LT.Z.AND.Z.LE.A(I+3)) J=K+6
      IF(A(I+3).LT.Z.AND.Z.LE.A(I+4)) J=K+8
      P(L)=B(J)+B(J+1)*Z
95    U(L)=B(J+1)
100   K=K+10
      IF(SAME.NE.0.) GO TO 120
      DO 110 I=2,NQD
      P(I)=P(1)
110   U(I)=U(1)
      IF(IY.NE.0) VL=VO
120   DO 125 I=1,NQDM
125   CCF(I)=PCF*SH(I)**1.5*(V(I)-V(I+1))/(P(I)*V(I)**2*.5*(R(I)-R(I+2)
     1 ))
```

```
      CCF(NQD)=PCF*SH(NQD)**1.5*(V(NQD)-VL)/(P(NQD)*V(NQD)*
     1 (R(NQD)-R(NQDP)))
      I=0
      DO 130 J=IW,NW,4
      I=I+1
      CCFO=PCF*SHO**1.5*(VO-V(1))/(P(1)*VO**2*(R(1)-R(2)   ))
      CFT=+CCF(I)-CCF(I-1)
      IF(I.EQ.1) CFT=+CCF(1)-CCFO
      IF(IY.NE.0) CCFO=0.0
      IF(IY.NE.0) CFT=-CFT
      PF=P(I)
      QHF=Q(I+1)
      QLF=Q(I)
      RF=R(I)
      SF=S(I)
      UF=U(I)
      IF(FF(I).LT.0.) RETURN
      AH(J)=DHX1(HF(I),FF(I),CF(I))
      HX=AH(J)/DX
      AS(J)=DCX(HF(I),FF(I),CF(I))
       AF(J)=DFX(HF(I),FF(I),CF(I))
 130  CONTINUE
      GO TO (132,135,136,138),IW
 132  IW=2
      X=XT+DX/2.
      IF(XM.GT.0.) R(1)=0.
      J=1
      D=2.
 133  DO 134 I=1,NQD
      CF(I)=AS(J)/D +C(I)
      FF(I)=AF(J)/D +F(I)
      HF(I)=AH(J)/D +H(I)
      TC=-CF(I)
      IF(IY.NE.0) TC=CF(I)
      IF((TC/PH+R(I)**2).LT.0.) RETURN
      S(I)=SQRT(TC/PH +R(I)**2)
      R(I+1)=S(I)
 134  J=J+4
      GO TO 25
 135  IW=3
      J=2
      GO TO 133
 136  IW=4
      X=XT+DX
      IF(XM.GT.0.) R(1)=0.
      J=3
      D=1.
      GO TO 133
 138  J=1
      DO 139 I=1,NQD
      C(I)=C(I)+(AS(J)+2.*AS(J+1)+2.*AS(J+2)+AS(J+3))/6.
      H(I)=H(I)+(AH(J)+2.*AH(J+1)+2.*AH(J+2)+AH(J+3))/6.
      F(I)=F(I)+(AF(J)+2.*AF(J+1)+2.*AF(J+2)+AF(J+3))/6.
      TC=-C(I)
      IF(IY.NE.0) TC=C(I)
      RT=TC/PH+R(I)**2
```

```
      IF(RT.GE.0.) GO TO 1385
      X=500.
      GO TO 139
1385  S(I)=SQRT(RT)
      R(I+1)=S(I)
139   J=J+4
1391  IXF=2
      X1=X*COS(TTH)
      WRITE(6,210) X,Z
      DO 140 I=1,NQDP
      IF(F(I).LT.O.) RETURN
      FS=SQRT(F(I))
      WRITE(6,180) C(I),H(I),SH(I),Q(I),FS ,CCF(I),R(I)
140   CONTINUE
      IF(Z.LT.AXX) GO TO 11
      RETURN
141   DO 142 I=1,NQD
      TC=-C(I)
      IF(IY.NE.0) TC=C(I)
      ZT=TC/PH+R(I)**2
      IF(ZT.LT.O.) GO TO 143
      S(I)=SQRT(ZT)
142   R(I+1)=S(I)
      GO TO 25
143   XM=1.
      AB=1.
      DO 144 I=1,NQD
      S(I)=S(I)*COS(TTH)
       R(I+1)=S(I)
144   C(I)=PH*(S(I)**2-R(I)**2)
      GO TO 11
147   WRITE(6,275)
      GO TO 5
150   FORMAT(I2)
155   FORMAT(5E15.8)
170   FORMAT(6E13.6)
180   FORMAT(3X,7E18.8)
210   FORMAT(///,2X,2HX=,2E15.8,/12X,1HC,17X,1HH,16X,2HSH,17X,1HQ,16X,
     1 3HSQF,16X,2HCF,16X,1HR)
250   FORMAT(     10X,2HH=,E15.8,1H+,E15.8,2H*X,//)
255   FORMAT(26X,8HIF Z LE ,E13.6,5H   P=,E13.6,1H+,E13.6,9H*Z AND U=,
     1 E13.6)
260   FORMAT(5X,2HIF,E13.6,14HLT Z AND Z LE ,E13.6,5H   P=,E13.6,1H+,
     1 E13.6,9H*Z AND U=,E13.6)
265   FORMAT(///,21X,1HX,19X,1HR,17X,5HTHETA,/,(10X,3E20.8))
270   FORMAT(1H1,25X,72HWE ARE WORKING FROM TRAILING EDGE OF CONE (X=0)
     1 TO REAR STAGNATION POINT,///)
275   FORMAT(1H1,35X,   56HWE ARE WORKING DOWNSTREAM OF REAR STAGNATION
     1 POINT (X=0),///)
      END
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13. ABSTRACT

The near wake of a cone in a hypersonic stream is analyzed by simultaneously solving the inviscid region and the viscous shear layer.

The inviscid region is solved by the use of rotational axisymmetric characteristics. It is assumed that viscosity and heat transfer play an important role only within a region bounded by streamlines which at the trailing edge of the cone are for the most part in the subsonic portion of the boundary layer. This region, termed the shear layer, lies between the Dividing Streamline (or centerline) and the Basic Streamline. The solution to the inviscid region is obtained by specifying conditions along the characteristic line originating at the shoulder of the cone, and by specifying the pressure distribution along a free surface (Basic Streamline) taken to be the streamline which at the shoulder of the cone separates the supersonic from the transonic and subsonic portions of the boundary layer. The pressure distribution along the Basic Streamline is iterated until the mass flow, momentum, and energy in the shear layer are consistent with the location of the Dividing Streamline and with the initial conditions at the edge of the cone.

Profiles for pitot pressure, static pressure and stagnation enthalpy are presented and compared with experiments at different downstream locations. The shape and strength of both the lip and recompression shock are also shown. Both sets of results are seen to be in very good agreement with the experimental results available.

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